

Algebraic K-Theory

Summer term 2011

Exercise sheet 5

Lück / Wegner

Exercise 13:

Let R be a commutative ring (with unit) and let G, H be groups.

- a) Show that the tensor product \otimes_R induces a bilinear map

$$m: K_0(RG) \times K_0(RH) \rightarrow K_0(R[G \times H]).$$

- b) Let C_* resp. D_* be a finitely dominated, projective RG - resp. RH -chain complex. Prove that $C_* \otimes_R D_*$ is a finitely dominated, projective $R[G \times H]$ -chain complex which satisfies $o(C_* \otimes_R D_*) = m(o(C_*), o(D_*))$.

Exercise 14:

Let X be a finitely dominated CW-complex.

Prove that $X \times S^1$ is homotopy equivalent to a finite CW-complex.

Exercise 15:

Let R be a principal ideal domain. Prove or disprove:

A positive, projective R -chain complex C_* is homotopy equivalent to a finite-dimensional R -chain complex if and only if there exists $N \in \mathbb{N}$ with $H_i(C_*) = 0$ for all $i > N$.

Please hand in your solutions at Christian Wegner's office (room 3.022) by **Monday, May 9th**. (Slide your solutions under the door if the room is locked.)

http://www.math.uni-bonn.de/people/wegner/lehre_SS2011/K-theory/