Advanced Topics in Topology:

Algebraic K-Theory

Summer term 2011

Exercise sheet 1 Lück / Wegner

Exercise 1:

Let R be a finite ring (with unit).

a) Show that the group homomorphism $\phi_R \colon \mathbb{Z} \to K_0(R)$ given by $1 \mapsto [R]$ is injective.

b) Check for $R = \mathbb{Z}/5\mathbb{Z}$ and $\mathbb{Z}/6\mathbb{Z}$ whether ϕ_R is surjective.

Exercise 2:

Let R be a commutative ring (with unit). Then every left R-module is automatically a right R-module as well, so that the tensor product of two left R-modules makes sense.

a) Show that the tensor product of two finitely generated projective modules is again finitely generated and projective.

b) Show that the tensor product makes $K_0(R)$ into a commutative ring with unit.

Exercise 3:

Let R be a ring (with unit). We denote by $P(R)$ the Grothendieck group of all countably generated projective R-modules, i.e., $P(R)$ is the abelian group with

- generators: isomorphism classes of countably generated projective R-modules,
- relations: $[P_1] - [P_2] + [P_3] = 0$ for every exact sequence $0 \to P_1 \to P_2 \to P_3 \to 0$ of countably generated projective R-modules.

Prove that $P(R)$ is the trivial group.

(Hint: Google "Eilenberg swindle".)

Please hand in your solutions at Christian Wegner's office (room 3.022) by **Monday, April 11th**. (Slide your solutions under the door if the room is locked.)

Exercise class: Wednesday, 12:15 - 14:00, room 0.011. We start next week (April 13th).

The following exercise sheets will be available every Tuesday at:
`http://www.math.uni-bonn.de/people/wegner/lehre_SS2011/K-theory/`