

# S4D3 Graduate Seminar on Advanced Geometry

**Organizers:** Ursula Hamenstädt and Robert Kucharczyk

**Friday 10h-12h, room 1.008**

**Organizational meeting: Monday, April 2, 14.15h in room 2.008**

**Topic: Tori**

Tori are important objects in virtually all areas of pure mathematics.

The goal of this seminar is to explore tori from various viewpoints with a leaning towards understanding important structures by studying the most important examples.

The program of the seminar will be determined according to the interests of the participants among the following topics.

- (1) **Differential geometry:** Tori as compact flat manifolds. Bieberbach's theorem. Tori as models for cusps in Riemannian manifolds of finite volume and strictly  $1/4$ -pinched negative curvature. Tori as compact totally geodesic embedded flat submanifolds in Riemannian manifolds of non-positive curvature. Tori as closed manifolds whose only metrics without conjugate points are flat.
- (2) **Low-dimensional topology:** Tori as decomposition surfaces for 3-manifolds. Tori as surgery surfaces for links. Tori as incompressible surfaces in non-compact 3-dimensional hyperbolic manifolds of finite volume.
- (3) **Dynamical systems:** Tori as phase spaces for Anosov diffeomorphisms. Tori as phase spaces for higher rank actions. Tori as deformation stable invariant structures for completely integrable systems (KAM theory).
- (4) **Complex analysis, algebraic geometry and number theory:** Tori as Riemann surfaces. The moduli space of tori. Tori as Jacobians of Riemann surfaces. Tori as projective cubics. Elliptic functions. Modular functions and modular forms. Special tori. Complex multiplication. Tori and rings of integers in imaginary quadratic number fields. Tori and Galois group actions. Kronecker's Jugendtraum (Kronecker's childhood dream).

**Prerequisites:** Everyone with solid knowledge in pure math equivalent to a bachelor degree is welcome.