University Bonn — WS 2017/18 Ursula Hamenstädt

Advanced Geometry I

Exercises 4 31. October 2017

Due to the holidays, this is 1/2 exercise sheet

4.1. Let $T^2 = \mathbb{C}/\Lambda$ be the quotient of the complex plane under a lattice $\Lambda = \{ae_1 + be_2 \mid a, b \in \mathbb{Z}\}$ where $e_1, e_2 \in \mathbb{C}$ are linearly independent over $\mathbb{R}$. The standard hermitian metric on $\mathbb{C}$ induces a hermitian metric on the holomorphic tangent bundle of T^2. Compute the curvature of the Chern connection for this metric.

4.2. Now let us assume for simplicity that $e_1 = 1, e_2 = i$. A holomorphic involution of a complex manifold M is a holomorphic map $\iota : M \to M$ such that $\iota^2 = \mathrm{Id}$.

(a) Show that the map $z \to -z$ is an involution of $\mathbb{C}$ which induces an involution of T^2, again denoted by ι.

(b) Show that the quotient T^2/ι has a natural structure of Riemann surface which is biholomorphic to $\mathbb{C}P^1$.

(c) Show that the pull-back of the tautological bundle over $\mathbb{C}P^1$ is a line bundle over $\mathbb{T}^2$. Does it have a nontrivial holomorphic section?