University Bonn — WS 2017/18	Ursula Hamenstädt
Advanced Geometry I	
Exercises 13	16. January 2018

13.1. Show that there is no compact Kähler manifold whose fundamental group is infinitely cyclic. (Hint: Remember what you learned about the de-Rham cohomology of Riemann surfaces).

13.2. Show that on a compact complex manifold M, every holomorphic differential form (this is a differential form of type $(p, 0)$ for some $p \geq 0$) is harmonic for every Kähler metric on M. (Hint: Use the full force of the Hodge theorem).

13.3. Let M be a compact Kähler manifold of dimension n and let $X \subset M$ be a commpact complex submanifold of dimension p. Define the *fundamental class* $[X] \in H^{n-p,n-p}(X, \mathbb{C})$ by

$$\int_M \alpha \wedge [X] = \int_X \alpha | X.$$

Show that $[X] \neq 0$.

13.4. Now assume that in Problem 13.3 M is a Riemann surface and X is one point p. Show that the class $[p]$ is defined by the curvature form of the line bundle $L(p)$ (defined by the divisor p) up to a constant. Show the same for arbitrary n and a compact complex submanifold of dimension $n - 1$.