7.1. Write down an explicit formula for the total Chern class of the tensor product of two complex vector bundles $E, F$ of rank two in terms of the Chern classes of $E, F$.

Show: The Chern classes of the tensor product of two arbitrary complex vector bundles $E, F$ are polynomials in the Chern classes of $E, F$.

7.2. Let $\hat{X} \to X$ be a two-fold covering with deck transformation $\tau : \hat{X} \to \hat{X}$. Show that $H^*(X; \mathbb{Z})$ can be identified with the fixed point set of $\tau^* : H^*(\hat{X}; \mathbb{Z}) \to H^*(\hat{X}; \mathbb{Z})$. Use this to show that for an integral domain $\Lambda$ containing $1/2$, the cohomology ring $H^*(G_{n}(\mathbb{R}^\infty); \Lambda)$ is a polynomial ring over $\Lambda$ generated by the Pontrjagin classes $p_1(\gamma^n), \ldots, p_{[n/2]}(\gamma^n)$.

7.3. i) Calculate the total Pontrjagin class of the tangent bundle of $S \times S'$ where $S, S'$ are closed oriented surfaces of genus $g, g'$.

ii) Let $k \in \mathbb{Z}$ be any number. Does there exist an oriented vector bundle over $S \times S'$ whose first Pontrjagin class equals $k$ times the preferred generator of $H^4(S \times S'; \mathbb{Z})$?

7.4. Calculate the first Pontrjagin class of the tangent bundle of $\mathbb{C}P^n$.

7.5. Quaternionic projective $n$-space $\mathbb{H}P^n$ is the space of all quaternionic lines in the $(n + 1)$-dimensional quaternionic vector space $\mathbb{H}^{n+1}$. Construct a tautological vector bundle $\gamma \to \mathbb{C}P^n$, show that it is naturally oriented and calculate its total Pontrjagin class.

7.6. Discussion problem: Is there a classifying space for quaternionic vector bundles? If yes, can you describe it and calculate its cohomology ring?