4.1. Verify the sign formula
\[ \tau_n \cup \tau_q = (1)^{nq} \tau_m \]
where \( n + q = m \), \( \tau_n \in H^q(\mathbb{R}^m, \mathbb{R}^m - (\mathbb{R}^n \times \{0\})) \), similarly for \( \tau_q \) and the orientation of \( \mathbb{R}^{m+q} \) is the sum of the orientations of \( \mathbb{R}^n \) and \( \mathbb{R}^q \) (in this order).

4.2. Let \( M^m \) be a smooth closed oriented manifold, \( N^{m-1} \subset M \) a closed submanifold of codimension one.
Show:
a) If \( N \) is orientable and \( 0 \neq \mu_N \in H_{m-1}(M, \mathbb{Z}) \) then \( \mu_N \) is indivisible.
b) If \( N \) is non-orientable then there is a smooth submanifold \( V \subset M \) diffeomorphic to the orientation cover of \( N \) which is contained in a tubular neighborhood of \( N \). Moreover, \( \iota_*\mu_V = 0 \in H_{m-1}(M, \mathbb{Z}) \) (here \( \iota : V \to M \) is the inclusion).

4.3. For each \( k \geq 1 \) construct a closed embedded oriented surface in \( \mathbb{C}P^2 \) which represents the homology class \( k\mathbb{C}P^1 \) (here \( \mathbb{C}P^1 \to \mathbb{C}P^2 \) is the standard embedding).

4.4. Calculate the intersection form on \( H_2 \) of
1) \( M = \mathbb{C}P^2 \# \mathbb{C}P^2 \) (means: Connected sum of \( \mathbb{C}P^2 \) with \( \mathbb{C}P^2 \) equipped with the opposite orientation). Deduce that \( M \) is not homotopy equivalent to \( S^2 \times S^2 \).
2) \( S_g \times S^1 \times S^1 \) where \( S_g \) is a closed surface of genus \( g \).
3) The product of a mapping torus \( S_g \times [0, 1]/ \sim \) for an orientation preserving diffeomorphisms with \( S^1 \).

4.5. Discussion problem: Is there an oriented vector bundle over an oriented manifold without nowhere vanishing section but with vanishing Euler class?