11.1. Show: The signature of a closed oriented 4-manifold which fibres over the circle vanishes.

11.2. Let \( M \to \pi \to B \) be a closed oriented 4-manifold which is a surface bundle over a closed oriented surface. Assume that the bundle has a section, i.e. that there is a smooth map \( \sigma : B \to M \) so that \( \pi \circ \sigma = \text{Id} \). Give an example of such a bundle with section so that \( \sigma(B) \) is not Poincaré dual to a multiple of the Euler class of the tangent bundle of the fibre.

11.3. Calculate the signature of the Grassmannian of oriented \( m \)-planes in \( \mathbb{R}^n \) for \((m, n) = (2, 4), (2, 6), (2, 8)\).

11.4. Choose numbers \( k_1, k_2, k_3 \in \mathbb{Z} \) and numbers \( n_1 < n_2 < n_3 \). Is there a multiplicative sequence \( \{K_n\} \) so that the \( K \)-genus of \( \mathbb{C}P^{n_i} \) equals \( k_i \)? What about the \( K \)-genus of \( \mathbb{C}P^{n_1}, \mathbb{C}P^{2n_1} \times \mathbb{C}P^{2n_2}, \mathbb{C}P^{2n_1} \times \mathbb{C}P^{2n_2} \times \mathbb{C}P^{2n_3} \)? Can you construct one for \( k_i = 2 \) and \( n_i = i \)?

11.5. Discussion problem: The intersection form of a four-manifold can be represented with respect to some basis by a symmetric matrix. Decide whether the following matrices arise and if you think the answer is yes, construct an example.

- a) Given any \( m \geq 0 \), the identity matrix of rank \( m \).
- b) Given \( m \geq 1, n \geq 1 \) the diagonal matrix with \( m \) diagonal elements one and \( n \) diagonal elements \(-1\).
- c) A matrix in block form whose first block is diagonal, with entries \pm 1, and an arbitrary number of blocks of the form

\[
\begin{pmatrix}
0 & 1 \\
1 & 0
\end{pmatrix}
\text{ or }
\begin{pmatrix}
0 & 1 \\
1 & -1
\end{pmatrix}.
\]