University Bonn — SS 2018	Ursula Hamenstädt
Advanced Geometry II	
Exercises 7	6. June 2018

7.1. Let M be obtained from $\mathbb{C}P^2$ by blowing up one point. Compute $\int_M c_1^2(M)$ and $\int_M c_2(M)$. Compute the integral of the Chern class of the canonical bundle over the exceptional curve.

7.2. Let S, S' be compact Riemann surfaces and let $f : S \to S'$ be an orientation preserving diffeomorphism. Show that $f^*c_1(S') = c_1(S)$. Can you construct an example of a pair M, M' of compact complex manifolds of complex dimension at least two with an orientation preserving diffeomorphism $f : M \to M'$ so that $f^*c(M') \neq c(M)$? (Here $c(M)$ is the total Chern class).

7.3. Let $S.S'$ be compact Riemann surfaces and let $f : S \to S'$ by a holomorphic map which is an unbranched covering. Show that $f^*c_1(S') = c_1(S)$.
Assume now that $f : S \to S'$ is a holomorphic two-sheeted branched cover with two simple branch points x, y (i.e. the differential of f has a simple zero at those points). Compute $f^*c_1(S')$.

7.4. Show that the product of two compact Riemann surfaces can not be embedded into $\mathbb{C}P^3$ (as a holomorphic submanifold).