University Bonn — SS 2018	Ursula Hamenstädt
Advanced Geometry II	
Exercises 2	18. April 2018

2.1. A *monoid* is a triple $(M, *, e)$ consisting of a set M and an operation

$$* : M \times M \to M,\ (a.b) \to a * b$$

and a distinguished element $e \in M$ with the following properties.

(a) Associativity: For $a, b, c \in M$, $(a * b) * c = a * (b * c)$.

(b) e is neutral: For all $a \in M$, $e * a = a * e = a$.

Show that presheaves and sheaves of abelian groups over a topological space X form a monoid. Denote the monoid of presheaves by $\mathcal{PS}(M)$.

2.2. A *submonoid* $U \subset M$ of a monoid is a subset which contains the neutral element and is closed under the operation $*$, i.e. if $u, v \in U$ then so is $u * v$. Let X, Y be topological spaces and let $F : X \to Y$ be a continuous map. Show that there is an induced map F^* of $\mathcal{PS}(Y)$ onto a submonoid of $\mathcal{PS}(X)$. Is the same true for sheaves? (Hint: Consider coverings).

2.3. Let X be a Riemann surface and denote by $\mathcal{C}(M)$ the sheaf of continuous $\mathbb{C}$-valued functions on M, by $\mathcal{C}^\infty(M)$ the sheaf of smooth $\mathbb{C}$-valued functions and by $\mathcal{O}(M)$ the sheaf of holomorphic functions. For a point $x \in X$, identify the stalks $\mathcal{C}_x(M)$, $\mathcal{C}_x^\infty(M)$ and $\mathcal{O}_x(M)$ of these sheaves at x. Show that the morphisms $\mathcal{O}(M) \to \mathcal{C}^\infty(M)$ and $\mathcal{C}^\infty(M) \to \mathcal{C}(M)$ induce homomorphisms of their stalks at x. Are these homomorphisms injective or surjective?

2.4. Now let X, Y be compact Riemann surfaces and let $F : X \to Y$ be a branched covering. Determine explicitly the sheaf of functions which is induced from the sheaf of holomorphic functions on Y via the map F and their stalks.