

SEMINAR S4D4  
GRADUATE SEMINAR ON ADVANCED GEOMETRY  
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## Simplicial volume and Mostow rigidity

The goal of the seminar is to give a geometric perspective on singular homology, with the celebrated Mostow rigidity theorem for finite volume hyperbolic manifolds as an application. The seminar is suitable for anyone who has some basic knowledge on algebraic topology and differential geometry.

The idea of simplicial volume is to endow (singular) homology of spaces and groups with a semi-norm and use this to construct invariants for groups and spaces. This construction endows the fundamental class of a closed manifold with a norm, and studying this norm reveals geometric information on the manifold.

It turns out that simplicial volume of a closed manifold only depends on the fundamental group of the manifold. In particular, simplicial volume vanishes for manifolds with amenable fundamental group. On the other hand, there is an explicitly computable positive number  $c(n) > 0$  only depending on the dimension  $n$  such that the simplicial volume of a closed hyperbolic manifold (that is, a closed Riemannian manifold  $M$  of constant curvature  $-1$ ) equals  $c(n)$  times the volume of  $M$ .

Following an idea of Gromov and Thurston, this fact can be used to give a proof of Mostow rigidity: For  $n \geq 3$ , two finite volume hyperbolic  $n$ -manifolds with isomorphic fundamental groups are isometric.

**Talks:**

- (1) Normed chain complexes, semi-norms on homology and cohomology; the criterion of Matsumoto and Morita for the semi-norm to be a norm.  
Frigerio pp 65-69.
- (2) Bounded cohomology of groups and comparison maps, and a criterion for injectivity of comparison maps for bounded cohomology of groups.  
Frigerio pp 1-5 and pp 70-71.
- (3) Bounded cohomology of groups and spaces and the translation principle between homology and cohomology.  
Frigerio pp 54-59, pp 71-72.
- (4) Simplicial volume and statement of the proportionality principle.  
Frigerio pp 78-79, pp 87-92.
- (5) Proof of the proportionality principle.  
Frigerio pp 92-97.
- (6) Simplicial volume of hyperbolic manifolds and the ideal boundary of a hyperbolic manifold.  
Frigerio pp 97-103, Benedetti-Petronio

- (7) Extensions of pseudo-isometries to the boundary.  
Benedetti-Petronio pp 84-93.
- (8) Mostow rigidity  
Benedetti-Petronio pp 121–127.

REFERENCES

- [BP92] R. Benedetti and C. Petronio, *Lectures on hyperbolic geometry*, Universitext, Springer, Berlin 1992.
- [F17] R. Frigerio, *Bounded cohomology and discrete groups*, AMS Math. Surveys and Monographs, AMS 2017.

Organizational meeting: Friday, July 15, at 12.15h in KIHS Wegelerstr. 10.