SEMINAR GROUP COHOMOLOGY AND BOUNDED COHOMOLOGY

The cohomology of a discrete group G is the cohomology of an explicit cochain complex defined by G, but it also is the cohomology of a classifying space for G.

Representations of groups yield group cohomology with coefficients. Group cohomology has many applications, most notably to the study of group actions on topological spaces.

Bounded cohomology is a variant of group cohomology turning groups and Banach modules over groups into graded semi-normed vector spaces. It can effectively be used to investigate asymptotic properties of groups.

The goal of the seminar is to give an introduction to group cohomology and bounded cohomology and discuss some topological applications.

References are:

(1) Kenneth Brown, Cohomology of groups, Springer graduate texts in Math. 87 (1982).
(2) Clara Löh, Group cohomology and bounded cohomology, an introduction for topologists, Course notes, Münster 2010.

All references beginning with a Roman number are taken from (1), the other references are taken from (2).

Lecture 1 - Definition and resolutions

goals: Definition of Group(co)homology via resolutions (II.3), also with non-trivial coefficients (III.1); Bar-resolution (I.5) und connection to classifying spaces (I.6).

Lecture 2 - Examples and low-dimensional groups 1

goals: Topological examples (II.4), Hopf-formula (Thm II.5.3)
complete literature: II.1-2, 4-5

Lecture 3 - Low-dimensional groups 2: Group extensions

goals: Classification of group extensions (IV.3.12; alternatively 1.4.4). Example: Euler-class and circle bundles.

Lecture 4 - Group actions on spheres

goals: Obstructions for groups which act on spheres (1.6)

Lecture 5 - Functorial viewpoint 1

goal: Functoriality of group cohomology (Prop. III.6.1), Lemma of Shapiro (Prop III.6.2)
Literature: III.5-6 oder 1.7.1-2

Lecture 6 - Functorial viewpoint 2

goal: Group cohomology via derived functors (III.7 oder 1.5)

Lecture 7 - Cohomological dimension

goal: Definition of cd (VIII.2), Theorem of Serre(VIII.3.1). Definition of FP_n– and FL–groups (VIII.5-6)
Literature: VIII.1-6

Lecture 8 - Cohomological dimension und topology

goal: Classifying spaces of the right dimension (VIII.7.1). The case of manifolds (VIII.8). Examples (VIII.9, in particular $SL_2(\mathbb{Z})$.
Literature: VIII.7-9

Lecture 9 - Poincaré duality groups

goal: Duality groups (VIII.10) and virtual dimension (VIII.11); outlook to recent results (discussed separately)
Literature: VIII.10-11.

Lecture 10-Bounded cohomology: Topologically

goal: Bounded cohomology of topological spaces
Literature: 2.4

Lecture 11-Bounded cohomology: Combinatorically

goal: Bounded cohomology of groups via the bar resolution
Literature: 2.5

Lecture 12-Amenable groups

goal: Characterization of amenable groups by bounded cohomology
Literature: 2.6