CANNON THURSTON MAPS AND MEASURES

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Abstract. We give a short proof of the following version of a recent result of Gadre, Maher, Pfaff and Uyanik: Consider a closed hyperbolic 3-manifold M which fibers over the circle, with fundamental group Γ and fiber a closed surface S. The measure class on the ideal boundary S^2 of hyperbolic 3-space defined by any Gibbs equilibrium state on the unit tangent bundle T^1M of M is singular with respect to the push-forward under a Cannon Thurston map of a measure class on the circle S^1 defined by a Gibbs equilibrium state on the unit tangent bundle T^1S of S.

1. Introduction

The fundamental group Γ of a closed hyperbolic 3-manifold which fibers over the circle contains a normal subgroup Γ_0 which is isomorphic to a surface group. The group Γ_0 is thus finitely generated, but exponentially distorted in Γ. In other words, the relation between the geometry of Γ_0 and the ambient group Γ is very complicated.

However, the groups Γ_0 and Γ are dynamically related in the following way. The group Γ_0 admits an embedding as a lattice into $\mathrm{PSL}(2,\mathbb{R})$, and via this embedding, it acts on the boundary S^1 of the hyperbolic plane $\mathbb{H}^2$. Similarly, the group Γ comes with a conjugacy class of an embedding into $\mathrm{PSL}(2,\mathbb{C})$ and hence it acts on the boundary S^2 of hyperbolic 3-space $\mathbb{H}^3$ as a group of Möbius transformations. By a result of Cannon and Thurston [CT07], these two actions are related: there exists a Γ_0-equivariant continuous surjective bounded-to-one map $F : S^1 \to S^2$ (see also [CL26] for more details on this construction).

While there are no Γ-invariant Radon measures on S^1 and S^2, we can ask for the relation between the push-forward under F of natural Γ_0-invariant measure classes on S^1 and natural Γ-invariant measure classes on S^2. Important examples of such invariant measure classes arise from exit measures of random walks on Γ_0, Γ, respectively. The main result of [GMPU26] is as follows.

Theorem 1 (Theorem 4 of [GMPU26]). *Suppose that M is a closed hyperbolic 3-manifold that fibers over the circle. Then the push-forward to S^2 under a Cannon-Thurston map of the hitting measure on S^1 of a random walk on Γ_0 defined by a probability measure whose support generates Γ_0 and is of finite exponential moment is singular to the hitting measure of a random walk on Γ defined by a probability measure whose support generates Γ and is of finite exponential moment.*

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Another family of invariant measure classes on S^1, S^2 arises from *symmetric Gibbs equilibrium states* on the unit tangent bundle of the manifolds $S = \Gamma_0\backslash\mathbb{H}^2$ and $M = \Gamma\backslash\mathbb{H}^3$. A Gibbs equilibrium state on T^1S is a probability measure which is invariant under the *geodesic flow* Φ^t and under the flip $v \to -v$. It corresponds to a *geodesic current*, that is, a Radon measure on the space $(S^1 \times S^1) \setminus \Delta$ of oriented geodesics on $\mathbb{H}^2$

which is invariant under Γ_0 and under the involution exchanging the two factors. Such a current is contained in the measure class of a product measure $\mu \times \mu$, and the measure class of μ is invariant under the action of Γ_0. Following [H02], we call the measure class of μ the *positive disintegration class* of the Gibbs measure. The same construction also applies to Gibbs equilibrium states on the unit tangent bundle T^1M of M.

By [CM07], any positive disintegration class on S^1, S^2 of a Gibbs equilibrium state can be obtained as the class of an exit measure of a random walk on Γ_0, Γ with finite first moment. Vice versa [K94], the exit measure of a random walk with finite first moment is a conditional measure for a Φ^t-invariant measure on T^S, T^1M. What is slightly less clear is the precise relation between a moment condition for the random walk and the regularity of a function defining the Gibbs state.

The goal of this note is to give a short proof of the following version of Theorem 1.

Theorem 2. *Under the assumption of Theorem 1, the push-forward under a Cannon Thurston map of the positive disintegration class of a Gibbs state on T^1S is singular to the disintegration class of any Gibbs state on T^1M.*

While the proof of Theorem 1 is geometric [GMPU26], the proof of Theorem 2 only uses dynamical tools, and it does not use any result from [GMPU26].

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2. The circle action of $\mathrm{Mod}(S_{g,1})$

Denote by $S_{g,1}$ a surface of genus $g \geq 2$ with one marked point (puncture), and by S_g the closed surface of genus g with no marked point. Let $\mathrm{Mod}(S_{g,1})$ and $\mathrm{Mod}(S_g)$ be the *mapping class group* of $S_{g,1}$ and S_g, respectively, which can be identified with the group of automorphisms and outer automorphisms of $\pi_1(S)$. These groups fit into the *Birman exact sequence*

$$0 \to \pi_1(S_g) \to \mathrm{Mod}(S_{g,1}) \xrightarrow{\Pi} \mathrm{Mod}(S_g) \to 0. \tag{1}$$

The *Teichmüller space* $\mathcal{T}(S_{g,1})$ of $S_{g,1}$ is the bundle over the Teichmüller space $\mathcal{T}(S_g)$ of marked hyperbolic structures on S_g whose fiber over a marked hyperbolic surface $X \in \mathcal{T}(S_g)$ is the universal covering of X, which is naturally identified with the hyperbolic plane $\mathbb{H}^2$. The mapping class group $\mathrm{Mod}(S_{g,1})$ acts properly discontinuously from the left on $\mathcal{T}(S_{g,1})$ as a group of bundle diffeomorphisms. The subgroup $\pi_1(S_g)$ preserves each of the fibers and acts cocompactly on each fiber. Thus $\pi_1(S_g)\backslash\mathcal{T}(S_{g,1})$ is a fiber bundle over $\mathcal{T}(S_g)$ whose fiber over a point X is just the hyperbolic surface X. In other words, this quotient is just the *universal curve* over $\mathcal{T}(S_g)$.

The unit tangent bundle $T^1\mathbb{H}^2$ of $\mathbb{H}^2$ is a $\pi_1(S_g)$-invariant circle bundle on the fiber of $\mathcal{T}(S_{g,1})$ over X. The union of these circle bundles defines a smooth circle bundle over $\mathcal{T}(S_{g,1})$ which is invariant under the action of $\pi_1(S_g)$ and descends to the *vertical unit tangent bundle* of the fibers of the universal curve. By naturality of this construction, equivalently by the Birman exact sequence, the circle bundle over $\mathcal{T}(S_{g,1})$ is in fact invariant under the whole mapping class group $\mathrm{Mod}(S_{g,1})$, and it descends to the vertical tangent bundle of the universal curve over the moduli space of pointed hyperbolic surfaces of genus g.

It was discovered by Morita [Mo88] that this circle bundle is *flat*, that is, there exists a homomorphism $\rho : \mathrm{Mod}(S_{g,1}) \to \mathrm{Homeo}(S^1)$ such that the circle bundle can be represented as $\mathrm{Mod}(S_{g,1})\backslash\mathcal{T}(S_{g,1}) \times S^1$ where the action of $\mathrm{Mod}(S_{g,1})$ on S^1 is via the homomorphism ρ. We next review the geometric construction of the homomorphism ρ as explained in Section 3 of [H21].

Let $D \subset \mathbb{C}$ be the unit disk, viewed as the Poincaré model of the hyperbolic plane $\mathbb{H}^2$. A marked hyperbolic surface $X \in \mathcal{T}(S_g)$ is defined by a discrete faithful homomorphism $\psi_X : \pi_1(S) \to \mathrm{PSL}(2,\mathbb{R})$. Thus X is the quotient of D by the action of the image group $\psi_X(\pi_1(S))$. Since $\mathrm{PSL}(2,\mathbb{R})$ acts simply transitively on the unit tangent bundle $T^1\mathbb{H}^2$ of $\mathbb{H}^2$, the choice of a unit tangent vector at $0 \in D$ determines an identification of $\mathrm{PSL}(2,\mathbb{R})$ with $T^1\mathbb{H}^2$.

Let $x \in X$ be the projection of $0 \in D$. The group $\mathrm{Mod}(S_{g,1})$ can also be identified with the group of isotopy classes of diffeomorphisms of S_g fixing the marked point $x \in X = S_g$, where the isotopy is required to fix x as well. In this way, the fiber subgroup $\pi_1(S)$ of $\mathrm{Mod}(S_{g,1})$ of the Birman exact sequence (1), consisting of inner automorphisms of $\pi_1(S)$, is identified with the *point pushing group*.

Any diffeomorphism of S_g fixing x is a bi-Lipschitz map for the hyperbolic structure X on S_g and hence it lifts to a $\pi_1(S_g,x)$-equivariant bi-Lipschitz map $\tilde{f} : \mathbb{H}^2 = D \to \mathbb{H}^2 = D$. Since f fixes x, this map can be chosen to fix 0. Such a bi-Lipschitz map maps geodesic rays starting at 0 to uniform quasi-geodesic rays, and any such quasi-geodesic ray is at uniformly bounded distance from a geodesic ray. As the circle $S^1 = \partial D$ can naturally be identified with the space of geodesic rays starting at 0, this construction then associates to a bi-Lipschitz homeomorphism f of X fixing x a homeomorphism $\rho(f)$ of S^1, and this homeomorphism only depends on the isotopy class of f fixing x. Moreover, it is Hölder continuous. This construction then defines the homomorphism $\rho : \mathrm{Mod}(S_{g,1}) \to \mathrm{Homeo}(S^1)$. Its restriction to $\pi_1(S_g) = \pi_1(X)$ is just the standard action of $\pi_1(X)$ on the ideal boundary of the universal covering $\mathbb{H}^2$ of X (see Section 3 of [H21]). If $\psi \in \mathrm{Mod}(S_{g,1}) = \mathrm{Aut}(\pi_1(S))$ is arbitrary, then ψ, viewed as an isotopy class of a diffeomorphism of X, conjugates the action of $\pi_1(X,x)$ on S^1 by the Hölder continuous homeomorphism $\rho(\psi)$.

From now on we denote by Γ_0 the fundamental group of the hyperbolic surface X, with universal covering $\mathbb{H}^2$. The standard identification of $T^1_0\mathbb{H}^2$ with the unit circle $S^1 = \partial\mathbb{H}^2$ maps the standard Lebesgue measure on $T_0\mathbb{H}^2$ to a Lebesgue measure λ on S^1. The measure class of λ is invariant under the action of Γ_0, and this action is ergodic. The following is well known.

Lemma 2.1. *If $\psi \in \mathrm{Mod}(S_{g,1})$ is such that $\Pi(\psi)$ has infinite order, then the homeomorphism $\rho(\psi)$ maps the Lebesgue measure λ on S^1 to a measure $\rho(\psi)_*\lambda$ on S^1 whose measure class is $\rho(\psi(\Gamma_0))$-invariant and ergodic, but it is singular with respect to λ.*

Proof. By [Bo88], the map which associates to a point $Y \in \mathcal{T}(S_g)$ in Teichmüller space the *geodesic current* for Γ_0 which is the *Liouville current* for the geodesic flow on the unit tangent bundle T^1Y of Y is an embedding.

The Liouville current of the basepoint X is a current in the measure class of $\lambda\times\lambda$. As two distinct points in $\mathcal{T}(S)$ determine singular currents and hence singular Lebesgue measures on S^1, if $\psi \in \mathrm{Mod}(S_{g,1})$ is such that the measures λ and $\rho(\psi)_*\lambda$ on S^1 are absolutely continuous, then the same holds true for the currents in the measure class of $\lambda\times\lambda$ and $\rho(\psi)_*\lambda\times\rho(\psi)_*\lambda$, respectively. But then we have $\Pi\psi(X) = X$ and $\Pi\circ\psi$ acts as an isometry on the hyperbolic surface X. Thus, as the isometry group of a hyperbolic surface is finite, if $\Pi(\psi)$ has infinite order, then the measures λ and $\rho(\psi)_*\lambda$ are singular as claimed. □

Lemma 2.1 can be extended in the following way. A *Gibbs measure* on T^1S is defined by Hölder continuous function f on T^1S, which we always assume to be normalized in such a way that its *pressure* $\mathrm{pr}(f) = \sup h_\nu + \int f d\nu$ vanishes. Here ν varies over all Φ^t-invariant probability measures on T^1S, and h_ν denotes the entropy of the measure μ. A Gibbs measure is absolutely continuous with respect to the strong stable and strong unstable foliation and hence it defines a Γ-invariant measure class μ_f on $S^1 = \partial\mathbb{H}^2$, called the *positive disintegration class* in the sequel. An example of a Gibbs measure is the Liouville measure. In the sequel we require all our Gibbs measures to be invariant under the *flip* $v \to -v$. This is equivalent to stating that the current they define is contained in the measure class of a product measure.

Proposition 2.2. *If the order of $\Pi(\psi)$ is infinite, then the measure classes of μ_f and $\rho(\psi)_*\mu_f$ are singular.*

Proof. Let $\psi \in \Gamma$ be such that $\Pi\psi \neq 0$. As explained in the beginning of this section, ψ induces a homeomorphism $S^1 \to S^1$ which conjugates the natural action of Γ_0, thought of as the subgroup of $\mathrm{PSL}(2,\mathbb{R})$ defining the base point $X \in \mathcal{T}(S)$, to the action of $\psi(\Gamma_0)$, thought of as the subgroup of $\mathrm{PSL}(2,\mathbb{R})$ defining $\psi(X)$. We denote this homeomorphism again by ψ. Then ψ maps the unique current ν_f for Γ_0 in the measure class of $\mu_f \times \mu_f$ to a $\psi(\Gamma_0) = \Gamma_0$-current in the measure class of $\psi(\mu_f) \times \psi(\mu_f)$.

Since the measure class of $\mu_f \times \mu_f$ is ergodic under the action of Γ_0, the current ν_f in the measure class of $\mu_f \times \mu_f$ is unique up to a constant multiple. Namely, any other current ξ in the same measure class as ν_f can be written as $\xi = h\nu_f$ where h is a Borel function on $(S^1 \times S^1) \setminus \Delta$ which is invariant under the action of Γ_0. By ergodicity of ν_f, this function is constant almost everywhere. As a consequence, if the measure classes of μ_f and $\psi_*\mu_f$ coincide, then up to a multiplicative constant, the currents ν_f and $\psi_*\nu_f$ are equal.

A Hölder continuous function $f : T^1S \to \mathbb{R}$ defining a Gibbs state whose associate geodesic current equals ν_f is not unique, but any two choices are *cohomologous*: They differ by a function of the form $Z(h)$ where h is a Hölder function whose restriction to a flow line of the geodesic flow Φ^t is differentiable, and $Z(h)$ denotes the derivative of h in direction of the flow. In particular, if the Hölder functions f, f' both define ν_f, then for any periodic orbit γ of Φ^t, the integral $\int_\gamma f = \int_\gamma f'$ where the orbit is equipped with the standard Φ^t-invariant Lebesgue measure.

Now the map $\psi : S^1 \to S^1$ is induced by a Hölder continuous orbit equivalence from T^1S, equipped with the geodesic flow of the metric X, onto T^1S, equipped with the geodesic flow of the metric $\psi(X)$. This orbit equivalence can be chosen to be a Hölder homeomorphism. As the push-forward $\psi_*f = f \circ \psi^{-1}$ under ψ of the Hölder function f defines the Gibbs state $\psi_*\nu_f = \nu_f$, the Hölder functions f and ψ_*f are cohomologous. This implies that for every periodic orbit γ for Φ^t, we have $\int_\gamma f = \int_{\psi(\gamma)} f$ where γ is thought of as a conjugacy class in Γ_0, and $\psi(\gamma)$ is its image under ψ, viewed as the periodic orbit defined by the image under ψ of this conjugacy class.

By the Nielsen-Thurston classification of mapping classes, if $\Pi\psi$ has infinite order then there exists a simple closed curve on S corresponding to a conjugacy class α in Γ_0 so that for all $\ell \neq k$, the conjugacy classes $\psi^\ell(\alpha)$ and $\psi^k(\alpha)$ are distinct. As a consequence, there is a number $m > 0$, and there are infinitely many pairwise distinct periodic orbits γ_j for the geodesic flow on T^1S so that $\int_{\gamma_j} f = m$ for all j. However, this is well known to be impossible. We provide a short argument.

Each of the periodic orbits γ_j is the unit tangent line of a closed geodesic in X. As For any $R > 0$ there are only finitely many geodesic in X of length at most R, we may assume that the orbits γ_j are ordered according to their increasing periods $\ell(\gamma_j) \leq \ell(\gamma_{j+1})$, with $\ell(\gamma_j) \to \infty$ as $j \to \infty$. Let $v_j \in \gamma_j$ and let η_j be the unique Φ^t-invariant Borel probability measure on T^1S supported on the periodic Φ^t-orbit of v_j; it is given by

$$\int h d\eta_j = \frac{1}{\ell(\gamma_j)} \int_0^{\ell(\gamma_j)} h(\Phi^t v_j) dt.$$

As $j \to \infty$, we know that $\int f d\eta_j \to 0$.

Take a weak limit η of a subsequence of the sequence of measures η_j, again denoted by η_j for simplicity. This is a Φ^t-invariant probability measure on T^1S. Since f is Hölder continuous, it holds

$$\int f d\eta = \lim_{j \to \infty} \int f d\eta_j = 0.$$

On the other hand, the pressure $\mathrm{pr}(f) = \sup\{h_\mu + \int f d\mu \mid \mu\}$ of f vanishes by assumption, and hence $h_\eta = 0$. Then η is the unique Gibbs equilibrium state for f. But this contradicts the fact that the entropy of a Gibbs state is positive.

This contradiction shows that indeed, the measure classes of μ_f and $\psi_*\mu_f$ are singular, completing the proof. □

3. Cannon Thurston maps

Consider now a closed hyperbolic 3-manifold M which fibers over S^1. The fundamental group Γ of M fits into an exact sequence

$$0 \to \pi_1(S_g) \to \Gamma \to \mathbb{Z} \to 0.$$

The monodromy of the fibration is pseudo-Anosov and hence the corresponding outer automorphism of $\pi_1(S_g)$ has infinite order. As a consequence, we can apply Proposition 2.2 to an element $\psi \in \Gamma$ which projects to a generator of $\mathbb{Z}$ and conclude.

Corollary 3.1. *If μ is the measure class on S^1 defined by a Gibbs equilibrium state on T^1S_g, then the measure classes μ and $\rho(\psi)_*\mu$ are singular.*

The group Γ is hyperbolic and quasi-isometric to hyperbolic 3-space and hence its Gromov boundary equals the two-sphere S^2. In spite of the fact that the subgroup $\Gamma_0 = \pi_1(S_g)$ of Γ is exponentially distorted, Cannon and Thurston showed that there is a *Cannon-Thurston map* $F : S^1 \to S^2$ [CT07]. The map F is continuous and equivariant with respect to the action of Γ_0 on S^1 (as the fundamental group of a hyperbolic surface) and the action of Γ_0 on S^2 (as a subgroup of Γ). Moreover, it is continuous, surjective and bounded-to-one (see [CL26] for a comprehensive account on Cannon Thurston maps).

We are now ready to show.

Proposition 3.2. *Let μ be the positive disintegration class on S^1 of a Gibbs equilibrium state on T^1S. Then the push-forward $F_*\mu$ of μ under a Cannon-Thurston map F is singular with respect to the positive disintegration class of any Gibbs measure on T^1M.*

Proof. Let as before $\psi \in \Gamma$ be an element whose projection to $\mathbb{Z}$ is a generator $\Pi(\psi)$ of the quotient $\mathbb{Z} = \Gamma_0\backslash\Gamma$ of Γ. The element ψ acts on Γ_0 as an automorphism which is not inner, and ψ acts on S^1 as a homeomorphism (where as before, we suppress the homomorphism ρ in our notation).

Consider the map $F \circ \psi : S^1 \to S^2$. It is continuous, surjective, bounded-to-one and equivariant with respect to the action of $\psi(\Gamma_0) = \Gamma_0$ and $\psi(\Gamma) = \Gamma$. Thus $F \circ \psi$ is a Cannon-Thurston map. As such a map is equivariant and fixed points of elements of Γ_0 are dense on S^1 and S^2, it is determined by the images of the fixed points of elements of Γ_0.

Let ζ be the positive disintegration class of the Gibbs equilibrium state of a Hölder continuous function on T^1M. An example is the standard Lebesgue measure on S^2. This measure class is invariant and ergodic under the action of Γ. Since the quotient $\Gamma_0\backslash\Gamma$ is infinite cyclic, it also is ergodic under the action of the normal subgroup Γ_0 [C01, H02]. Thus the Γ_0-invariant measure classes $F_*\mu$ and ζ either coincide or are singular.

Now if these measure classes coincide, then by Γ-invariance of the measure class of ζ, the measure class $F\circ\psi_*\mu$ equals the measure class of $F_*\mu$. But by Proposition 2.2, the measure classes $\mu,\psi_*\mu$ are singular. Moreover, the map F is bounded-to-one, and there exists a Borel subset A of S^1 whose complement is of measure zero for both $\mu,\psi_*\mu$ and such that the restriction of F to A is injective.

Namely, consider the equivalence relation $\sim$ on S^1 defined by $x\sim y$ if and only if $F(x)=F(y)$. Following [CL26], this relation is the union of two disjoint (no perfect fits) *laminar* relations in the sense of Definition 1.16 of [CL26]: Its equivalence classes are closed subsets of S^1, the relation is closed in the space of unordered distinct pairs of points, and distinct equivalence classes are *unlinked*. As the relation is invariant under the action of Γ_0, this can be thought of as follows. Given any two points $x\neq y\in S^1$ with $F(x)=F(y)$, the geodesic in $S=\Gamma_0\backslash\mathbb{H}^2$ which is the projection of the geodesic in $\mathbb{H}^2$ with endpoints x,y is *simple*, that is, it does not have self-intersections.

On the other hand, if μ is the positive disintegration class of a Gibbs equilibrium state ν, then μ-almost every $z\in S^1$ is the endpoint of a geodesic whose tangent line projects to a Birkhoff regular orbit for the geodesic flow in T^1S, equipped with the Gibbs measure ν. We claim that such a point can not be the endpoint of a lift of a simple geodesic on S.

To see this assume otherwise. Then there are geodesics $\gamma_1,\gamma_2\subset\mathbb{H}^2$ with the same endpoint $z\in S^1$ so that the tangent line γ_1' of γ_1 is generic for the measure ν, and the projection of the geodesic γ_2 to S is simple. Now for suitable parameterizations of γ_1,γ_2 by arc length and the Sasaki metric d_S on $T^1\mathbb{H}^2$ it holds

$$d_S(\gamma_1'(t),\gamma_2'(t))\leq Ce^{-t}$$

for a constant $C>0$, and the same holds true for the projections of γ_1',γ_2' to T^1S.

Let $P:\mathbb{H}^2\to X$ be the canonical projection. Then since $P(\gamma_2)$ is simple, for any $\alpha>0$ there exists a number $t(\alpha)>0$ so that for any $t>t(\alpha)$ with the property that $\gamma_1(t)\in P(\gamma_2)$, the angle between $\gamma_1'(t)$ and γ_2' is at most α.

On the other hand, as $\gamma'(0)$ is a Birkhoff regular point for the geodesic flow Φ^t on T^1S and the Gibbs measure ν, the forward Φ^t-orbit of $(P\gamma_1)'(t(\alpha))$, which is just the tangent line $(P\gamma_1)'[t(\alpha),\infty)$, is dense in T^1S. For sufficiently small α we arrive at a contradiction.

Applying the same reasoning to $\psi_*\mu$ yields that there exists a Γ_0-invariant Borel subset B of S^1 with the following properties.

(1) $S^1\setminus B$ is of measure zero for both $\mu,\psi_*\mu$.
(2) The restriction of F to B is injective.

As μ and $\psi_*\mu$ are singular and $F|B$ is injective, the measure classes of $F_*\mu$ and $F_*(\psi_*\mu)$ are singular as well and hence they are singular to the positive disintegration measure of any Gibbs equilibrium state on T^1M. $\square$

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