Chow Rings of Irreducible Symplectic Varieties
(Beauville’s Weak Splitting Property)

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Main Theorem

The main result presented here, which is called the weak splitting property under certain conditions and gives an explicit geometric proof, is a significant contribution to the study of Chow rings of irreducible symplectic varieties.

Definition.

For an irreducible symplectic variety $X$, denote its Kähler cone by $K_X$. Define its birational Kähler cone as

$$
\bar{K}_X = \bigcup \{ \bar{f}(x) : x \in H^2(X, \mathbb{R}) \},
$$

where the union is taken over all birational maps $f : X \to Y$ from $X$ to another irreducible symplectic variety $Y$. Denote it by $\bar{K}(X)$ if $X$ is an irreducible symplectic variety.

Conjecture 5: Let $X$ be an irreducible symplectic variety. Suppose that there exists a birational map $f : X \to Y$ with $\bar{K}(X)$ as the intersection of the image of $f$ in $\bar{K}(Y)$ and all irreducible components of the fibers of $f$.

The following is the main result of this paper: (It can be found in [Rie14a]:

Theorem 6: For an irreducible symplectic variety $X$, there is a weak splitting property.

The advantage of this result is that Conjecture 5 has been intensively studied during the last few years. A long series of conjectures was proved by M. Kontsevich, S. Kontsevich, and more recently by M. Kontsevich and S. L. Kudryashov.

Main Ingredients of the Proof

The first step is an observation by Huybrechts (in [Huy15]): based on a result of Huybrechts determining relations in cohomology. In order to prove the weak splitting property, it is enough to show:

$$
0 \in CH^3(X)(0) = 0 \pmod{\text{dim}(X)}.
$$

Use Huybrechts’ description of $\overline{K}(X)$; Mukhin’s result that certain cohomological relations are necessary operators, and a refined version of Theorem 6 along with various computations, to reduce to the following:

$$
0 \in CH^3(X)(0) = 0 \pmod{\text{dim}(X)}.
$$

This follows from Conjecture 5, using the fact that the base of the Lagrangian fibration is $X$.

References


