Matrix Lie groups

1) Basic definitions
examples ($O(n,\mathbb{C})$, $SL(n,\mathbb{C})$, compact symplectic group
Poincaré group, ...

2) Topological properties
Compactness, Connectedness, simply connectedness
$SO(3,\mathbb{R})$
Homomorphism of m.L.g.

3) Matrix exponential

$$e^X = \sum \frac{X^m}{m!} \qquad X \in M_n(\mathbb{C})$$

$\log A$

Every inv $n \times n$ matrix is of the form e^X

4) Lie algebras: $G \longrightarrow \mathrm{Lie}(G) = \mathfrak{g}$

examples

Lie ~~alg~~ Group homom. $\longleftrightarrow$ Lie alg. homomorphisms

Complexifications

5) The exponential map again

interpret $\exp: \mathfrak{g} \to G$

6) Representations

def. / examples

Corresp $\mathrm{Rep}(G) \longleftrightarrow \mathrm{Rep}(\mathfrak{g})$

unitary repr

$\otimes$-products . .

7) Complete reducibility

Unitary repr are completely reducible

every repr of a compact matrix Lie group is completely reducible.

8) Groups vs Lie algebras

study this

main example: $SO(3)$, $SU(2)$, $\mathfrak{sl}(2,\mathbb{C})$

9) Campbell-Baker-Hausdorff formula

formula $\log(e^X e^Y)$ $\qquad \mathfrak{g} \leftrightarrow G$

10) $\mathfrak{g}$ vs G (follow up of talk 9)

univ covers $\longleftrightarrow$?

subgroups

11) Compact Lie groups

Tori, Weyl groups $N_G(T)/T = W$

Any two maximal tori are conjugate

quotients

12) Weyl integration formula $\int_G f(x)\,dx = \cdots$

Structure of Weyl groups W

13) Representations of G

"weights" (π, V)

Character of a representation $\mathrm{trace}(\pi(x))$

$$\int \mathrm{trace}(\pi(x)) \, \overline{\mathrm{trace}(\pi'(x))} = \begin{cases} 1 & V = W \\ 0 & \text{else} \end{cases}$$

V W

14) Weyl character formula

$$\chi_\pi(e^H) = \frac{\sum \ldots}{\ldots}$$

15) Reconstructing representations

Does there exist for every dom. int. weight a repr. of G?