Graduate Seminar on Representation Theory

Hopf algebras and tensor categories

Winter term 2019/2020

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Categories of modules/comodules of Hopf algebras are among the most important examples of monoidal categories (or tensor categories). After a short introduction to monoidal categories and Hopf algebras we will focus on two famous constructions and results.

Tannakian formalism. Representations of algebraic groups or (equivalently) comodules of commutative Hopf algebras define tensor categories with many amenable properties: abelian, Hom-spaces are finite dimensional k-vector spaces, objects have duals,... Such tensor categories are called Tannaka categories. A theorem of Saavedra, Deligne-Milne and Deligne asserts that in fact every Tannaka category is the representation category of a suitable pro-algebraic group (at least if k is algebraically closed).

Reshetikhin-Turaev knot invariants. For finite dimensional vector spaces $V_1 \otimes V_2 \cong V_2 \otimes V_1$. A generalization of such a property leads to the notion of a braided monoidal category. If such a category also has (left) duals and a family of twists, it is called a ribbon (tensor) category. Reshetikhin and Turaev defined for any object in a ribbon category a knot (or link) invariant. These invariants are however trivial unless the braiding is non-symmetric. It turns out that it is very difficult to construct any such non-symmetric ribbon categories.

Prerequisites: Good knowledge of categorial/functorial language. It is helpful, though not strictly required, to have some knowlegde of algebraic groups and representation theory. For the last talks on quantum groups the speaker should be familiar with the standard theory of semisimple Lie algebras.

Organizational meeting: Friday, July 5th, 4.15 pm in seminar room 0.006.

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