Thorsten Heidersdorf
Universität Bonn

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Exercises for the lecture Algebra 1
—Exercise sheet 3—

Exercise 1 (10 points). (Finitely generated) Show:

(i) If $f : M \to R^n$ is a surjective R-linear map and M finitely generated, then $ker(f)$ is finitely generated.

(ii) Free submodules of finitely generated modules are finitely generated

(iii) If there exists a finitely generated submodule N of a module M such that M/N is finitely generated, then M is finitely generated.

Exercise 2 (5 points). (Quotient fields) Let R be an integral domain which is not a field. Then the quotient field $Quot(R)$ is not finitely generated as an R-module.

Exercise 3 (5 points). (Local-Global ?) Let M be an R-module. If M_p is a finitely generated R_p-module for all prime ideals p in R, is M finitely generated as an R-module?

Exercise 4 (10 points). (Exactness and Hom) Let R be a ring and

$$0 \longrightarrow N_1 \longrightarrow N_2 \longrightarrow N_3$$

a sequence of R-modules. Show the equivalence of the following statements:

(i) The sequence is exact.

(ii) For all R-modules M

$$0 \longrightarrow \mathrm{Hom}_R(M, N_1) \longrightarrow \mathrm{Hom}_R(M, N_2) \longrightarrow \mathrm{Hom}_R(M, N_3)$$

is exact.

Due date: Thursday, 25.04.2019, before the start of the lecture at 2.15pm.