

1 RHA, SS 13, Exercise Sheet 12

Due June 17, 2013.

Exercise 1:

Write down the proof of Brouwer's fixed point theorem discussed in the lecture: a smooth map from the closed ball to itself has a fixed point.

Exercise 2 :

Let $\Omega = [0, 1]$ and $f : \Omega \rightarrow \mathbf{R}^n$ continuously differentiable. Prove that the set of critical values

$$\{y \in \mathbf{R} : \exists x \in \Omega : f(x) = y, f'(x) = 0\}$$

has Lebesgue measure zero.

Hint: cover the set of critical values by intervals whose inverse images contain intervals with much larger measure. Use a covering argument to make these intervals disjoint.

Exercise 3:

Prove the n -dimensional version of Exercise 3.

Let $\Omega \subset \mathbf{R}^n$ and $f : \Omega \rightarrow \mathbf{R}^n$ continuously differentiable. Prove that the set of critical values

$$\{y \in \mathbf{R}^n : \exists x \in \Omega : f(x) = y, \nabla f(x) = 0\}$$

has Lebesgue measure zero.