Prof. Dr. Stefan Schwede
Dr. Sven van Nigtevecht
Wintersemester 2026/27

Hauptseminar Homotopietheorie S2D4
'Einfache Homotopietheorie'

Dienstags 14:15 Uhr, Seminarraum N 0.008 (Neubau)

In this seminar, we study the notion of a *simple homotopy equivalence* using Whitehead groups. We follow Chapter 11 of the lecture notes by Davis–Kirk [DK01] and the short paper by Stöcker [Stö70]. Further, more extensive references include Milnor's survey article [Mil66] and Cohen's book [Coh73]; these also contain ample further references.

In the last part of the seminar, we discuss the closely related subject of *Wall's finiteness obstruction*. This is an invariant of a space that detects whether it has the homotopy type of a finite CW-complex. We will mostly follow Pedersen's account [Ped17]; further references for this part include the survey articles by Mislin [Mis95] and Ferry–Ranicki [FR00], as well as Wall's original papers [Wal65; Wal66].

Both themes are briefly touched upon in Rosenberg's book [Ros94], which puts them in a broader context.

Instructions Each talk should be at most 80 minutes long, accounting for questions and comments. Talks may be given either in English or in German, depending on the speaker's preference. It is up to each presenter to choose exactly what should be presented from each topic, although the main theorems and definitions should always be given. Make sure to include plenty of examples (even if not explicitly asked for in the abstracts below), context, and motivation.

You should schedule a meeting with Sven van Nigtevecht (`nigtevec@math.uni-bonn.de`) to discuss the contents and structure of your talk. This meeting should take place *at least two weeks* before your talk.

1. The Whitehead group of a CW-complex (H. Gemein) 13.10.2026

Cover [Stö70, Sections 1–2], as well as [DK01, Definition 11.3 (2)] and the example following it.

2. The groups $K_1(R)$ and $Wh(R)$ (M. Clausen) 20.10.2026

Cover [DK01, Section 11.2]; also cover (briefly) the relative version $K_1(R, I)$ from [Ros94, Section 2.5], and mention the exact sequence from [Ros94, Theorem 2.5.4].

3. The torsion of a chain complex (F. Mayer) 27.10.2026

Cover [DK01, Section 11.3] up until (and including) Definition 11.22.

4. Whitehead torsion for CW-complexes (A. Palabiyik) 03.11.2026

Cover the rest of [DK01, Section 11.3], as well as parts (1) and (2) of [DK01, Theorem 11.31].

5. Whitehead torsion detects simple homotopy equivalences (T. Elshof) 10.11.2026

Cover [Stö70, Section 3] and part (3) of [DK01, Theorem 11.31]. More specifically: Use [Coh73, (7.4)] to prove the results from [Stö70, Section 3]. The last part of [DK01, Theorem 11.31] then follows using [Stö70, Satz 3] and [Coh73, (5.8)].

6. Homotopy classification of 3-dimensional lens spaces (A. Sonnenberg) 17.11.2026

Cover [DK01, Section 11.6] up until (and including) Theorem 11.35.

7. Simple homotopy classification of 3-dimensional lens spaces 24.11.2026

Cover the rest of [DK01, Section 11.6], in particular [DK01, Theorem 11.37]. If time allows, state (but do not prove) [DK01, Theorem 11.38].

8. Finitely dominated spaces (A. Palabiyik) 01.12.2026

Introduce finitely dominated spaces and the finiteness question to which Wall's obstruction will provide the answer; see [Ros94, Introduction to Section 1.7] and [FR00, Definition 1.1, Proposition 1.5]. Then state [Ped17, Theorem 1.3] and cover the first half of the proof (i.e., up until and including the proof of Lemma 1.9). For this, introduce geometric realisation [HM22, Section 2.2].

9. Wall's finiteness obstruction I (A. Tchagalidze) 08.12.2026

Cover the second half of the proof of [Ped17, Theorem 1.3] (i.e., starting from Lemma 1.11). Then define $\widetilde{K}_0$ [FR00, Section 2] and give the construction of the finiteness obstruction for chain complexes [Ros94, Theorem 1.7.12].

10. Wall's finiteness obstruction II (A. Tchagalidze) 15.12.2026

Define Wall's obstruction [Ped17, Remark 1.4] and prove Wall's Theorem [Ped17, Theorem 1.2 (4), (5)]. If time allows, discuss the example of H-spaces with infinite fundamental group [Ped17, Proposition 2.2].

References

[Coh73] M. M. Cohen. *A Course in Simple-Homotopy Theory*. Red. by P. R. Halmos. Vol. 10. Graduate Texts in Mathematics. Springer New York, 1973. DOI: 10.1007/978-1-4684-9372-6.

[DK01] J. Davis and P. Kirk. *Lecture Notes in Algebraic Topology*. Vol. 35. Graduate Studies in Mathematics. American Mathematical Society, 12th July 2001. DOI: 10.1090/gsm/035.

[FR00] S. Ferry and A. Ranicki. *A Survey of Wall's Finiteness Obstruction*. 9th Aug. 2000. arXiv: math/0008070. Preprint.

[HM22] G. Heuts and I. Moerdijk. *Simplicial and Dendroidal Homotopy Theory*. Vol. 75. Springer International Publishing, 2022. DOI: 10.1007/978-3-031-10447-3.

[Mil66] J. Milnor. 'Whitehead Torsion'. *Bulletin of the American Mathematical Society* 72.3 (1966), pp. 358–426. DOI: 10.1090/S0002-9904-1966-11484-2.

[Mis95] G. Mislin. 'Wall's Finiteness Obstruction'. In: *Handbook of Algebraic Topology*. Elsevier, 1995, pp. 1259–1291. DOI: 10.1016/B978-044481779-2/50027-4.

[Ped17] E. K. Pedersen. 'Wall's Finiteness Obstruction'. *Journal of Pure and Applied Algebra* 221.7 (July 2017), pp. 1691–1698. DOI: 10.1016/j.jpaa.2016.12.023.

[Ros94] J. Rosenberg. *Algebraic K-Theory and Its Applications*. Red. by S. Axler, F. W. Gehring and P. R. Halmos. Vol. 147. Graduate Texts in Mathematics. Springer New York, 1994. DOI: 10.1007/978-1-4612-4314-4.

[Stö70] R. Stöcker. 'Whiteheadgruppe topologischer Räume'. *Inventiones Mathematicae* 9.4 (Dec. 1970), pp. 271–278. DOI: 10.1007/BF01425481.

[Wal65] C. T. C. Wall. 'Finiteness Conditions for CW-Complexes'. *The Annals of Mathematics* 81.1 (Jan. 1965), p. 56. DOI: 10.2307/1970382.

[Wal66] C. T. C. Wall. 'Finiteness Conditions for CW Complexes. II'. *Proceedings of the Royal Society of London. Series A. Mathematical and Physical Sciences* 295.1441 (29th Nov. 1966), pp. 129–139. DOI: 10.1098/rspa.1966.0230.