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NAKAYAMA AND GENERALIZED NAKAYAMA CONJECTURE

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Let K be a field, and let A be a finite-dimensional K-algebra.

1. Nakayama Conjecture

Let
$$0 \to {}_AA \to I_0 \to I_1 \to \cdots$$
be a minimal injective resolution of the regular representation of A.

Conjecture 1.1 (Nakayama Conjecture). *If I_j is projective for all $i \geq 0$, then A is selfinjective.*

2. Generalized Nakayama Conjecture

Again, let
$$0 \to {}_AA \to I_0 \to I_1 \to \cdots$$
be a minimal injective resolution of the regular representation of A.

Conjecture 2.1 (Generalized Nakayama Conjecture [AR]). *For each indecomposable injective A-module I there exists some $j \geq 0$ such that I is isomorphic to a direct summand of I_j.*

The Generalized Nakayama Conjecture implies the Nakayama Conjecture.

The above conjecture is equivalent to the following conjecture, see [AR]. To be more precise, Conjecture 2.1 holds for all finite-dimensional K-algebras A if and only if Conjecture 2.2 holds for all finite-dimensional K-algebras A.

Conjecture 2.2. *Let S be a simple A-module. Then there exists some $i \geq 0$ such that* $\mathrm{Ext}_A^i(S, {}_AA) \neq 0$.

Here is an even stronger conjecture:

Conjecture 2.3. *Let M be a non-zero A-module. Then there exists some $i \geq 0$ such that* $\mathrm{Ext}_A^i(M, {}_AA) \neq 0$.

References

[AR] M. Auslander, I. Reiten, *On a generalized version of the Nakayama conjecture*, Proc. Amer. Math. Soc. 52 (1975), 69–74.

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