9. Übungsaufgaben Darstellungstheorie I, WS 06/07

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1. Let $R = K[X, Y]$ be the polynomial ring in two (commuting) variables. Show:

- R is not local;
- 0 and 1 are the only idempotents in R;
- The Jacobson radical of R is 0.

2. A module V is called **local** if it contains a maximal submodule U, which contains all proper submodules of V. Show: If V is local, then V contains exactly one maximal submodule. Construct an example which shows that the converse is not true.

3. Show: Every module of finite length is a sum of local submodules.

4. Let V be a module of length n. Show: V is semisimple if and only if V cannot be written as a sum of $n-1$ local submodules.

5. Find the original references for Schreier's Theorem, and also for the Jordan-Hölder Theorem.

6. Let V be a module of finite length. Let f be a non-zero endomorphism of V such that for all non-zero endomorphisms g of V we have $l(\mathrm{Im}(f)) \leq l(\mathrm{Im}(g))$. Show: $f^2 = 0$ and $\mathrm{Im}(f)$ is indecomposable.