8. Übungsaufgaben Darstellungstheorie I, WS 06/07

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1. Use the Converse Bottleneck Lemma to show that for $n \geq 2$ the short exact sequences $0 \to N(1) \xrightarrow{\iota_1} N(2) \xrightarrow{\pi_2} N(1) \to 0$ and

$$0 \to N(n) \xrightarrow{\left[\begin{smallmatrix}\iota_n \\ \pi_n\end{smallmatrix}\right]} N(n+1) \oplus N(n-1) \xrightarrow{[\pi_{n+1}, -\iota_{n-1}]} N(n) \to 0$$

are Auslander-Reiten sequence in $\mathcal{N}^{\text{f.d.}}$.

2. Determine all composition series of the 2-module $V = (K^5, \phi, \psi)$ where

$$\phi = \begin{bmatrix} c_0 & & & & \\ & c_0 & & & \\ & & c_1 & & \\ & & & c_2 & \\ & & & & c_3 \end{bmatrix} \quad \text{and} \quad \psi = \begin{bmatrix} 0 & 0 & 1 & 0 & 1 \\ & 0 & 0 & 1 & 1 \\ & & 0 & & \\ & & & 0 & \\ & & & & 0 \end{bmatrix}$$

with pairwise different elements c_0, c_1, c_2, c_3 in K.

3. Let V_1 and V_2 be modules, and let S be a factor of a filtration of $V_1 \oplus V_2$. Show: If S is simple, then there exists a filtration of V_1 or of V_2 which contains a factor isomorphic to S.

4. Construct indecomposable modules V_1 and V_2 with $l(V_1) = l(V_2) = 2$, and a filtration of $V_1 \oplus V_2$ containing a factor T of length 2 such that T is not isomorphic to V_1 or V_2.