7. Übungsaufgaben Darstellungstheorie I, WS 06/07

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1. Let

$$0 \to U \xrightarrow{f_1} V_1 \xrightarrow{g_1} W \to 0$$

and

$$0 \to U \xrightarrow{f_2} V_2 \xrightarrow{g_2} W \to 0$$

be equivalent short exact sequences of J-modules, and let $a\colon U \to X$ be a homomorphism. Show that the two short exact sequences $a_*(f_1, g_1)$ and $a_*(f_2, g_2)$ are equivalent.

2. Let

$$0 \to U \xrightarrow{f} V \xrightarrow{g} W \to 0$$

be a short exact sequence of J-modules, and let $a\colon U \to X$, $a'\colon X \to X'$, $b\colon Y \to W$, $b'\colon Y' \to Y$ be homomorphisms of J-modules. Show:

- The induced sequences $(a'a)_*(f, g)$ and $a'_*(a_*(f, g))$ are equivalent;
- The induced sequences $(bb')^*(f, g)$ and $(b')^*(b^*(f, g))$ are equivalent;
- The induced sequences $a_*(b^*(f, g))$ and $b^*(a_*(f, g))$ are equivalent.

3. Construct an example of two non-equivalent short exact sequences

$$0 \to U \to V_1 \to W \to 0$$

and

$$0 \to U \to V_2 \to W \to 0$$

of J-modules with $V_1 \cong V_2$.

4. Let $\mathcal{M} = \mathcal{M}(1)^{\text{f.d.}}$ be the module category of 1-modules (V, ϕ) with V finite-dimensional. Construct the Auslander-Reiten quiver of $\mathcal{M}$. (Hint: Remember how we constructed the Auslander-Reiten quiver of the module category $\mathcal{N}^{\text{f.d.}}$ of all 1-modules (V, ϕ) with V finite-dimensional and ϕ nilpotent.)