## 5. Übungsaufgaben Darstellungstheorie I, WS 06/07

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**1.** Let

$$\begin{array}{ccccccccc} 0 & \longrightarrow & V_1 & \xrightarrow{f_1} & V & \xrightarrow{f_2} & V_2 & \longrightarrow & 0 \\ & & & & \downarrow a & & & & \\ 0 & \longrightarrow & W_1 & \xrightarrow{g_1} & W & \xrightarrow{g_2} & W_2 & \longrightarrow & 0 \end{array}$$

be a diagramm of $J$-modules with exact rows.

Show: There exists a homomorphism $a_1 \colon V_1 \to W_1$ with $af_1 = g_1 a_1$ if and only if there exists a homomorphism $a_2 \colon V_2 \to W_2$ with $g_2 a = a_2 f_2$.

**2.** Let

$$\begin{array}{ccccccccc} V_1 & \xrightarrow{f_1} & V_2 & \xrightarrow{f_2} & V_3 & \xrightarrow{f_3} & V_4 & \xrightarrow{f_4} & V_5 \\ \downarrow a_1 & & \downarrow a_2 & & \downarrow a_3 & & \downarrow a_4 & & \downarrow a_5 \\ W_1 & \xrightarrow{g_1} & W_2 & \xrightarrow{g_2} & W_3 & \xrightarrow{g_3} & W_4 & \xrightarrow{g_4} & W_5 \end{array}$$

be a commutative diagramm of $J$-modules with exact rows.

Show: If $a_1$ is an epimorphism, and if $a_2$ and $a_4$ are monomorphisms, then $a_3$ is a monomorphism.

If $a_5$ is a monomorphism, and if $a_2$ and $a_4$ are epimorphisms, then $a_3$ is an epimorphism.

If $a_1, a_2, a_4, a_5$ are isomorphisms, then $a_3$ is an isomorphism.

**3.** Let $0 \to U \xrightarrow{f} V \xrightarrow{g} W \to 0$ be a short exact sequence of $J$-modules.

Show: The exact sequence $(f, g)$ splits if and only if for all $J$-modules $X$ the sequence

$$0 \to \mathrm{Hom}_J(X, U) \xrightarrow{\mathrm{Hom}_J(X,f)} \mathrm{Hom}_J(X, V) \xrightarrow{\mathrm{Hom}_J(X,g)} \mathrm{Hom}_J(X, W) \to$$

is exact. (By the results we obtained so far, it is enough to show that $\mathrm{Hom}_J(X, g)$ is surjective for all $X$.)

**4.** Recall: For any partition $\lambda$, we defined a 1-module $N(\lambda)$. Let

$$0 \to N(n) \xrightarrow{f_1} N(2n) \xrightarrow{g_1} N(n) \to 0$$

be a short exact sequence with $f_1 = {}^t[1, 0]$ and $g_1 = [0, 1]$, and let

$$\eta : \quad 0 \to N(n) \xrightarrow{f_2} N(\lambda) \xrightarrow{g_2} N(n) \to 0$$

be a short exact sequence with $\lambda = (\lambda_1, \lambda_2)$.

Show: There exists some homomorphism $a \colon N(n) \to N(n)$ such that $a_*(f_1, g_1) = \eta$.

**5.** Construct an example of a short exact sequence

$$0 \to U \to U' \oplus W \to W \to 0$$

such that $U \not\cong U'$.