5. Übungsaufgaben Darstellungstheorie I, WS 06/07

****

1. Show that the 2-module $(K[T], \frac{d}{dT}, T\cdot)$ is simple and that $K \cong \mathrm{End}(K[T], \frac{d}{dT}, T\cdot)$ if $\mathrm{char}(K) = 0$.

Compute $\mathrm{End}(K[T], \frac{d}{dT}, T\cdot)$ if $\mathrm{char}(K) > 0$.

Show that $(K[T], \frac{d}{dT}, T\cdot)$ is not simple in case $\mathrm{char}(K) > 0$.

For endomorphisms f and g of a vector space let $[f, g] = fg - gf$ be its *commutator*.

Show that $[T\cdot, \frac{d}{dT}] = 1$.

2. Let (V, ϕ_1, ϕ_2) be a 2-module with $V \neq 0$ and $[\phi_1, \phi_2] = 1$.

Show: If $\mathrm{char}(K) = 0$, then V is infinite dimensional.

Hint: Assume V is finite-dimensional, and try to get a contradiction. You could work with the *trace* (of endomorphisms of V). Which endomorphisms does one have to look at?

3. Let $V = (V, \phi_j)_{j\in J}$ be a finite-dimensional J-module such that all ϕ_j are diagonalizable.

Show: If $[\phi_i, \phi_j] = 0$ for all $i, j \in J$ and if V is simple, then V is 1-dimensional.

4. Let U be a submodule of a module V.

Prove that

$$(U + \mathrm{rad}(V))/U \subseteq \mathrm{rad}(V/U).$$

Prove that $U \subseteq \mathrm{rad}(V)$ implies that $\mathrm{rad}(V/U) = \mathrm{rad}(V)/U$.