

4. Übungsaufgaben Darstellungstheorie I, WS 06/07

1. Show that every simple 1-module is finite-dimensional.

Show: If K is algebraically closed, then every simple 1-module is 1-dimensional.

Show: If $K = \mathbb{R}$, then every simple 1-module is 1- or 2-dimensional.

2. Let $A = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \in M(2, \mathbb{C})$. Show that there does not exist a matrix $B \in M(2, \mathbb{C})$ such that (\mathbb{C}^2, A, B) is simple.

3. Show: If V is a 2-dimensional module with at least 5 submodules, then every subspace of V is a submodule.

4. Let

$$V = (K^3, \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}).$$

Compute $\text{End}(V)$ (as a set of 3×3 -matrices).

Determine all idempotents e in $\text{End}(V)$.

Determine all direct sum decompositions $V = V_1 \oplus V_2$ (with drawings in case $K = \mathbb{R}$).

Describe the map $e \mapsto (\text{Bild}(e), \text{Kern}(e))$ (where e runs through the set of idempotents in $\text{End}(V)$).

5. Let

$$V = (K^3, \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}).$$

Show: $\text{End}(V)$ is the set of matrices of the form

$$\begin{pmatrix} a & 0 & 0 \\ b & a & 0 \\ c & 0 & a \end{pmatrix}$$

with $a, b, c \in K$.

Use this to show that V is indecomposable.

Show that V is not simple.

6. Let V and W be J -modules. We know that $V \times W$ is again a J -module.

Let $f: V \rightarrow W$ be a module homomorphism, and let

$$\Gamma_f = \{(v, f(v)) \mid v \in V\}$$

be the *graph* of f .

Show: The map $f \mapsto \Gamma_f$ defines a bijection between $\text{Hom}_J(V, W)$ and the set of submodules $U \subseteq V \times W$ with $U \oplus (0 \times W) = V \times W$.