3. Übungsaufgaben Darstellungstheorie I, WS 06/07

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1.

Classify all submodules U of $V = N(2,1), N(3,1), N(2,2)$ and determine in each case the isomorphism class of U and of the factor module V/U.

For $K = \mathbb{F}_2$ and $K = \mathbb{F}_3$ draw the corresponding Hasse diagrams.

Let $K = \mathbb{F}_p$ with p a prime number, and let λ and μ be partitions. How many submodules U of V with $U \cong N(\lambda)$ and $V/U \cong N(\mu)$ are there?

2.

Let K be a field of characteristic 0. For integers $i, j \in \mathbb{Z}$ with $i \leq j$ let $M(i,j)$ be the module (K^{j-i+1}, Φ, Ψ) where

$$\Phi = \begin{pmatrix} i & & & & \\ & i+1 & & & \\ & & \ddots & & \\ & & & j-1 & \\ & & & & j \end{pmatrix} \text{ and } \Psi = \begin{pmatrix} 0 & 1 & & & \\ & 0 & 1 & & \\ & & \ddots & \ddots & \\ & & & 0 & 1 \\ & & & & 0 \end{pmatrix}.$$

Compute $\operatorname{Hom}(M(i,j), M(k,l))$ for all integers $i \leq j$ and $k \leq l$.

3.

Let K be an algebraically closed field. For $m \geq 1$ an m-module is by definition a J-module where $J = \{1, \dots, m\}$.

Classify the simple 1-modules (V, ϕ).

Classify the 2-dimensional simple 2-modules (V, ϕ, ψ).

For every $n \geq 1$ construct an n-dimensional simple 2-module (V, ϕ, ψ).