9. Übungsaufgaben Darstellungstheorie II, SS 07

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1. Let A be a finite dimensional K-algebra. Let Γ_A be its Auslander-Reiten quiver and let d_A be the corresponding valuation. Prove Lemma 6.31 from the lectures: Let $\mathcal{C}$ be a connected component of (Γ_A, d_A) such that

$$\mathcal{C} = \bigcup_{n\geq 0}(\mathcal{C} \cap {}_n\underline{\Delta}) = {}_\infty\underline{\Delta} \cap \mathcal{C}.$$

Then $\mathcal{C}$ is a preprojective component of (Γ_A, d_A).

2. Show for a finite-dimensional algebra A the following are equivalent:

(i) A is connected;
(ii) If $A = A_1 \times A_2$ then $A_1 = 0$ or $A_2 = 0$;
(iii) 0 and 1 are the only central idempotents in A;
(iv) For any indecomposable projective A-modules P and P' there exists a tuple $(P_1, P_2, \dots P_m)$ of indecomposable projective modules such that $P_1 = P$, $P_m = P'$ and for each $1 \leq i \leq m-1$ we have $\mathrm{Hom}_A(P_i, P_{i+1}) \oplus \mathrm{Hom}_A(P_{i+1}, P_i) \neq 0$;
(v) For any simple A-modules S and S' there exists a tuple $(S_1, S_2, \dots, S_m)$ of simple modules such that $S_1 = S$, $S_m = S'$ and for each $1 \leq i \leq m-1$ we have $\mathrm{Ext}^1_A(S_i, S_{i+1}) \oplus \mathrm{Ext}^1_A(S_{i+1}, S_i) \neq 0$.

3. Knit the preprojective component of (Γ_A, d_A) for the $\mathbb{R}$-algebra

$$A = \begin{pmatrix} \mathbb{R} & \mathbb{C} & \mathbb{C} \\ 0 & \mathbb{C} & \mathbb{C} \\ 0 & 0 & \mathbb{R} \end{pmatrix}.$$

4. Let A be a finite dimensional K-algebra. Prove the following:

(i) A vertex $[X]$ in (Γ_A, d_A) is a sink vertex if and only if X is a simple projective A-module;
(ii) The quiver Γ_A contains no loops.

5. Let Q be any quiver without loops and K any field. Describe the simple projective KQ-modules.