2. Übungsaufgaben Darstellungstheorie I, WS 06/07

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Aufgabe 1 Let W and U_i, $i \in I$ be a set of submodules of a module $(V, \phi_j)_j$ such that for all $k, l \in I$ we have $U_k \subseteq U_l$ or $U_k \supseteq U_l$. Show that

$$\sum_{i\in I} U_i = \bigcup_{i\in I} U_i$$

and

$$\bigcup_{i\in I}(W \cap U_i) = W \cap \left(\bigcup_{i\in I} U_i\right).$$

Aufgabe 2 Let K be a field and let $V = (K^4, \phi, \psi)$ be a module such that

$$\phi = \begin{pmatrix} \lambda_1 & & & \\ & \lambda_2 & & \\ & & \lambda_3 & \\ & & & \lambda_4 \end{pmatrix}$$

with pairwise different $\lambda_i \in K$. How can the lattice of submodules of V look like?

Aufgabe 3 Which of the following lattices can be the lattice of submodules of a 4-dimensional module of the form (V, ϕ, ψ)? In each case you can work with a field K of your choice. Of course it is better if you find examples which are independent of the field, if this is possible.