7. Übungsaufgaben Darstellungstheorie II, SS 07

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1. Prove the following lemma: Let X be a finitely generated A-module and Y any A-module. Let $f\colon X \to Y$ be any homomorphism, then the following are equivalent:

(i) f factors through a projective module;
(ii) f factors through a finitely generated projective module;
(iii) f factors through a free module of finite rank.

2. Let

$$0 \longrightarrow X \xrightarrow{f} Y \xrightarrow{g} Z \longrightarrow 0$$

be a short exact sequence of A-modules. Prove that the following are equivalent:

(i) g is right almost split, and X is indecomposable;
(ii) f is left almost split, and Z is indecomposable;
(iii) f and g are irreducible.

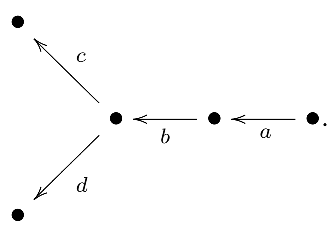
(Hint: Use results from Skript 1, in particular the Converse Bottleneck Lemma.)

3. Let Q be the quiver

$$A_n : \qquad 1 \longrightarrow 2 \longrightarrow \cdots \longrightarrow (n-1) \longrightarrow n.$$

Find all the indecomposable KQ-modules and calculate τM for each indecomposable module M.

4. Let Q be the quiver



Let A be the algebra $KQ/(ba)$. Find the indecomposable A-modules and for each indecomposable A-module M calculate τM.