

6. Übungsaufgaben Darstellungstheorie II, SS 2007

1. Let A be a finite-dimensional algebra. Show that M is a finitely generated projective module if and only if $\text{Tor}_1^A(A/\text{rad } A, M) = 0$.

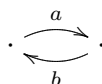
2.

Let A be a finite-dimensional algebra. Let Tr denote the transpose map. Let M and N be A -modules. Prove the following properties of Tr :

- (i) $\text{Tr}(M \oplus N) \cong \text{Tr}(M) \oplus \text{Tr}(N)$;
- (ii) $\text{Tr}(M) = 0$ if and only if M is projective;
- (iii) $\text{Tr}(M)$ has no non-zero projective direct summands;
- (iv) M is an indecomposable nonprojective module if and only if $\text{Tr}(M)$ is indecomposable and non-projective;
- (v) If M and N are indecomposable and non-projective, then $\text{Tr}(\text{Tr}(M)) \cong M$, and we have $M \cong N$ if and only if $\text{Tr}(M) \cong \text{Tr}(N)$.

3.

Let Q be the quiver



Determine the indecomposable modules for $KQ/(ab)$ and for each indecomposable M construct $D\text{Tr}(M)$ where $D = \text{Hom}_K(-, K)$ is the duality functor.