1. Übungsaufgaben Darstellungstheorie II, WS 06/07

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1. Let Q be the quiver

Determine the indecomposable submodules of the indecomposable projective modules of $A := KQ$ and of $B := KQ/I$ where I is generated by $da - ec$. What about arbitrary path algebras?

2. Let $A := KQ/I$ where Q is a quiver with vertices $1, \ldots, n$ and I is an admissible ideal. By $e_1, \ldots, e_n$ we denote the paths of length 0 in Q. Let $\mathrm{D} := \mathrm{Hom}_K(-, K)$ be the "duality functor". What is an injective module? (Find the definition somewhere.) Show that $\mathrm{D}(e_i A)$ is an indecomposable injective A-module and show that all finite-dimensional indecomposable injective A-modules are of this form. How do the modules $\mathrm{D}(e_i A)$ look like (in terms of paths in the quiver Q)?

3. Let Q be the quiver

and let $A := KQ/I$ where I is generated by $ba - dc$ and fe. For every simple A-module S_i, $1 \leq i \leq 6$ construct an exact sequence

$$0 \to P(t) \xrightarrow{f_t} \cdots \xrightarrow{f_2} P(1) \xrightarrow{f_1} P(0) \xrightarrow{f_0} S_i \to 0$$

such that $\mathrm{Ker}(f_j)$ is small in $P(j)$, and $P(j)$ is projective for all $j \geq 0$. (Note that the modules $P(j)$ are not necessarily indecomposable.)

4. Find the original references for Kaplansky's Theorem and for the Krull-Remak-Schmidt-Azumaya Theorem. Can the Crawley-Jønnson-Warfield Theorem be generalized? Try to find a discussion of this question somewhere in the literature.