12. Übungsaufgaben Darstellungstheorie I, WS 06/07

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1. Let Q be a quiver. Show that KQ is finite-dimensional if and only if Q has no oriented cycles.

2. Let $V = (K \xleftarrow{1} K \xrightarrow{1} K)$ and $W = (K \xleftarrow{1} K \rightarrow 0)$ be representations of the quiver $1 \leftarrow 2 \rightarrow 3$. Show that $\mathrm{Hom}_Q(V, W)$ is one-dimensional, and that $\mathrm{Hom}_Q(W, V) = 0$.

3. Let Q be the quiver

$$1 \rightarrow 2 \rightarrow \cdots \rightarrow n.$$

Show that KQ is isomorphic to the subalgebra

$$A := \{M \in M_n(K) \mid m_{ij} = 0 \text{ if there is no path from } j \text{ to } i\}$$

of $M_n(K)$.

4. Let Q be any quiver. Determine the centre of KQ. (Reminder: The **centre** $C(A)$ of an algebra A is defined as $C(A) = \{a \in A \mid ab = ba \text{ for all } b \in A\}$.)

5. Let Q be a quiver with n vertices. Show that there are n isomorphism classes of simple KQ-modules if and only if Q has no oriented cycles.

6. Let Q be a quiver. Show that the categories $\mathrm{Rep}_K(Q)$ and $\mathrm{Mod}(KQ)$ are equivalent.