# 10. Übungsaufgaben Darstellungstheorie I, WS 06/07

**********************************************************************

**1.** Let $V = (K[T], T\cdot)$. Show:

(a): The direct summands of $V \oplus V$ are 0, $V \oplus V$ and all the submodules of the form

$$U_{f,g} := \{(hf, hg) \mid h \in K[T]\}$$

where $f$ and $g$ are polynomials with greatest common divisor 1.

(b): There exist direct summands $U$ of $V \oplus V$ such that none of the modules 0, $V \oplus V$, $U_{1,0} = V \oplus 0$ and $U_{0,1} = 0 \oplus V$ are a direct complement of $U$ in $V \oplus V$.

**2.** Let $M_1, \dots, M_t$ be pairwise non-isomorphic modules of finite length, and let $m_i \geq 1$ for $1 \leq i \leq t$. Define

$$V = \bigoplus_{i=1}^{t} M_i^{m_i},$$

and let $R = \operatorname{End}(V)$ be the endomorphism ring of $V$. Show: There exists an idempotent $e$ in $R$ such that $e(V)$ is isomorphic to $\bigoplus_{i=1}^{t} M_i$, and we have $R = ReR$.

**3.** Let $A = K\langle X_1, \dots, X_n\rangle$ be the $K$-algebra of polynomials in $n$ non-commuting variables $X_1, \dots, X_n$, and let $J = \{1, \dots, n\}$. Show: The category of $J$-modules is equivalent to $\operatorname{Mod}(A)$.

**4.** Let $A$ be a $K$-algebra. Show that the category of left $A$-modules is equivalent to the category of right $A^{\mathrm{op}}$-modules.