Theoretical Background

In this project we make use of a mixture of different research approaches and theories including for example:

Community of Practice is a term which originates in a general learning theory by Lave & Wenger [11]. A community of practice is a group of people with a common interest in a certain domain. The establishment strategies for further development or the attainment of knowledge related to this domain can be goals of such a community of practice. Wenger’s theory considers learning rather as a social act than as an individual process. Another crucial assumption within this framework is that learning is a life long process and belongs to the human nature as much as for example eating or sleeping. According to Wenger, a community of practice consists of three strongly related components: mutual engagement, joint enterprise and shared repertoire.

Action Research (AR) is a specific branch in the field of educational design research (DR). This field can be divided according to their research purpose into validation and development studies. AR belongs to the latter one: The purpose of the developmental studies is “to develop research-solutions for complex problems in educational practice” and it is “defined as the systematic analysis, design and evaluation of educational interventions with the dual aim of generating research-based solutions for complex problems in educational practice, and advancing our knowledge about characteristics of these interventions and the processes of designing and developing them” [9, p. 36]. The driving force of AR are the participating teachers who want to develop their own teaching practice (“change through action”).

The characteristic of design research is its dual-use: design research is an established way for intertwining theory and practice and thus finding a way for new knowledge for both contexts. On the one hand AR can develop easier usable knowledge for practitioners (in comparison to only theoretical research). On the other hand it can develop easier usable knowledge for practitioners (in comparison to only theoretical research). The teachers argued that the lack of symmetry is only a minor problem and many recognized the need for a self-regulated aspect of the pupils. If the pupils are aware of their own misconceptions, they can develop easier usable knowledge for practitioners. On the other hand the obtained results have already been backed up by the process in which they were obtained (proof by process) [2], thus yielding a scientific justification for the findings. This research method can be illustrated in the developmental cycle below.

The Math-il.de Project

A group with a few teachers which meets monthly was founded in November 2015. The communication especially between the meetings has been supported by the internet platform math-il.de which was specially designed for this kind of research. The way of proceeding can be sketched as follows:

This platform was established in 2009 on the initiative of Rainer Kaenders [6].

Course of events:
1. Schoolbook analysis: The usual treatment in schoolbooks often misses to emphasize two fundamental aspects of symmetry: its versatility and its power in problem-solving situations. Symmetry is mostly not used as a tool.
2. Initial Question: Is it possible to implement a teaching practice that imparts basic ideas of higher mathematics by taking the example of symmetry?
3. Discussion: The teachers argued that the lack of symmetry is only a minor problem and they identified poor manipulation skills as a more urgent problem in their own classes. Maybe symmetry can help to simplify the manipulations of algebraic expressions.
4. Test: A small diagnostic test was conducted where the main focus lied on solving brackets with a minus sign in front, e.g. $-3(x + y) + 2x - (2p - x)$. Conclusions (for our small sample):
   - Pupils can solve the brackets in isolated cases.
   - If the expressions become longer and more complex, the pupils will make more mistakes.
   - Varying representations of terms is difficult for the pupils, in particular, verbal and geometric expressions cause troubles.

Similar conclusions have been drawn earlier in standard references (e.g. [8]).

5. Literature research: Which important educational publications do exist on manipulation of algebraic expressions? What is meant by structure of an expression?
6. State of play: After the literature research, it was agreed that the following features/aspect should be included in the development of the learning environments:
   - Verbally simplifying terms
   - Critical attitude towards manipulation of terms [8]
   - Manipulation of terms can reveal (non-)dependencies of different variables [8]
   - Substitution of variables
   - Different representations of terms: What does symmetry mean in these representations?

Selected Educational Publications on Algebraic Expressions

There are many publications on structure of algebraic expressions. However, different authors emphasize various aspects of structure. In the following we briefly describe important publications in this area:

- Kieran [7]: Based on a notion of structure as “the arrangement of the parts in a whole, the aggregate of elements of an entity to which relationships are attributed” Kieran also distinguishes these two dimensions in structure of an algebraic or arithmetic expressions:
  - Surface structure (disposition) describes the particular calculation represented by the term respectively its internal order (normal form). They refine structure sense (proposed by Linchevski & Livneh) as the fluent handling of familiar structures. In [5], they suggest a teaching unit in which the pupils learn to identify five classical structures in highschool algebra.
  - Systemic structure (relational) can be seen as the class of all equivalent terms (respectively equations) for a given term (respectively equation).
- Hock & Dreyfus [4] assign to each algebraic expression a shape which reveals (possibly after a transformation) its internal order (normal form). They refine structure sense (proposed by Linchevski & Livneh) as the fluent handling of familiar structures. In [5], they suggest a teaching unit in which the pupils learn to identify five classical structures in highschool algebra.
- Malle [8] defines the recognition of a structure of an algebraic expression differently, namely as the identification of subexpressions, e.g.
  
  $\frac{a \times b}{c} + 2 = 11, \quad \frac{a + b}{c} - 11, \quad \frac{a - b}{c} + 11, \quad \frac{a \times b}{c} + 2$.

- Rüede [10]: Contrasting the above mentioned point of views in which structure is an objective property of the given algebraic expression, Rüede also takes the individual person into account. By structuring he refers to the relations in an algebraic expression constructed (resp. perceived) by an individual person. Although not explicitly mentioned, Rüede takes up again Freudenthal’s perspective “mathematics as a human activity” [1].
- Henningsdóttir-Hellesen & Reet [5] recently pointed out, that although many studies have been made about different approaches for the introduction of algebraic notation, it is still not known whether these approaches lead to a substantial improvement of the algebraic abilities of pupils. Moreover, they mention that there is a lack of practical teaching suggestions for the topics manipulation of terms and equations.
- Freudenthal [1] claims that substitutions of variables go short in most algebra classes resulting in problems in the structuring of single algebraic expressions.

Outline of the Project

The group agreed on developing two learning environments appropriate for our needs.

Managing a Chocolate Fabric (small groups in regular class, 8th grade): Based on different algebraic terms (also in text and geometric format), they must take different decisions. From there on, in every lesson (= round) new aspects arise which complicate the situation. Now and more detailed parameters lead to substitutions and the resulting longer terms can be simplified again.

Repetition-Class (mid-August, around 10 pupils): The main focus of this class is that the pupils adopt a more critical attitude towards their own activity of manipulation of algebraic expression. This will be mainly supported by exercises in which it is necessary to justify/verify every single step.

References


