PROGRAM FOR THE ARBEITSGEMEINSCHAFT ON

"CLASSICAL RESULTS IN ALGEBRAIC AND DIFFERENTIAL TOPOLOGY"

(WINTER TERM 2012/2013)

Introduction

Other than in previous years, this term's *Arbeitsgemeinschaft* will not cover a single topic, but will rather have sessions with different subjects and varying organizers.

Contents

Date: November 23, 2012.

Session 1: Algebraic K-theory of finite fields (October 11, 2012) organized by Steffen Sagave

In his influential work [Qui72], Quillen computed the algebraic K-theory groups of finite fields $\mathbb{F}_q$ with q elements. This is a very important calculation of higher K-theory groups which (according to Quillen) partly motivated the definition of higher algebraic K-theory.

The strategy of Quillen's proof is to show that the algebraic K-theory space of $\mathbb{F}_q$ is homotopy equivalent to a certain space $F\Psi^q$ that can be defined in terms of an Adams operation. The homotopy groups of $F\Psi^q$ can easily be computed and give the desired K-theory groups.

Talk 1: Adams operations and the cohomology of $F\Psi^q$ (Karol Szumilo). The first talk should start with a review of Adams operations and the definition of $F\Psi^q$. After the (simple) computation of the homotopy groups of $F\Psi^q$, the speaker should give the computation of the cohomology and homology of $F\Psi^q$.
References: [Qui72, §2 - §6] and [Ada62].

Talk 2: The Brauer lifting and $K(\mathbb{F}_q)$ (Marcus Zibrowius). The second talk should begin with a construction of the Brauer lifting. This can be used to compute $\bigoplus_n H_*(\mathrm{GL}_n(\mathbb{F}_q))$. Moreover, the Brauer lifting induces an isomorphism $H_*(B\mathrm{GL}(\mathbb{F}_q)) \to H_*(F\Psi^q)$. After a brief review of the plus construction, this isomorphism provides the main step in the computation of $K_*(\mathbb{F}_q)$.
References: [Qui72, §7 - §12]

References for Session 1

[Ada62] J. F. Adams, *Vector fields on spheres*, Ann. of Math. (2) **75** (1962), 603–632. MR0139178 (25 #2614)

[Qui72] Daniel Quillen, *On the cohomology and K-theory of the general linear groups over a finite field*, Ann. of Math. (2) **96** (1972), 552–586. MR0315016 (47 #3565)

Session 2: Smoothing theory I (October 25, 2012) organized by Tibor Macko and Martin Olbermann

The aim of these two sessions is to elucidate the statement and to indicate the proof of the main theorem of smoothing theory whose one version is as follows.

Theorem 0.1. *Let M be a* PL*-manifold. Then M admits a smooth structure if and only if its stable "*PL *tangent microbundle" reduces to a vector bundle.*

Microbundles were invented by Milnor who also showed that they possess a lot of familiar properties of vector bundles. In particular, there exists a classifying space for n-dimensional PL-microbundles denoted $\mathrm{BPL}(n)$. Also, it turns out that n-dimensional vector bundles give rise to n-dimensional PL-microbundles and this is reflected in the forgetful map $\mathrm{BO}(n) \to \mathrm{BPL}(n)$. Also the PL-microbundles can be stabilized analogously to the stabilization for vector bundles. Hence existence of the reduction in the theorem can be stated homotopy theoretically as the existence of a lift

$$\begin{array}{ccc} & & \mathrm{BO} \\ & \nearrow & \downarrow \\ M & \xrightarrow[\tau_M]{} & \mathrm{BPL} \end{array}$$

The main technical tool in the proof of Theorem 0.1 is the Product Structure Theorem (PST):

Theorem 0.2. *Let M be a* PL*-manifold and $s \geq 1$. Then the set concordance classes of smooth structures on M is in bijection with the set of concordance classes of smooth structures on $M \times \mathbb{R}^s$.*

The first two talks cover the PST. The third and fourth talk will be about microbundles, their classifying spaces and about the proof of the main theorem using the PST.

There are various sources available for this theory. There are also different versions of the above statements. We hope to elucidate some of these differences in the lectures as well. The published references are the books [HM74] and [KS77] and the paper [Mil64]. However, we find the notes [Lur09] very useful and readable. Therefore we develop the program mainly along these notes, which we believe are sufficient for the purposes of this AG.

Talk 1: Definitions and Product Structure Theorem I. (Martin Olbermann). Follow [Lur09, Lectures 19-20]. At the beginning take plenty of time to discuss carefully the necessary definitions, analogously to [HM74, pages 7-9]. Also [HM74, pages 10-12] has a nice discussion of concordance versus isotopy versus diffeomorphism.

The proof of the PST is by considering the PL projection map $pr_2 : M \times \mathbb{R} \to \mathbb{R}$ for some smooth structure on $M \times \mathbb{R}$. Define the PD analogue of a smooth submersion and show that if pr_2 has this property, the PST follows. Show that pr_2 satisfies the "submersion" property away from an isolated set of points. Sketch the proofs of [Lur09, Lecture 19, Lemma 7 and Propositions 8] quickly (instead of working with epsilons). You should at least reduce the PST to [Lur09, Lecture 20, Problem 5].

Talk 2: Product Structure Theorem II. (Moritz Rodenhausen.) This talk should cover [Lur09, Lectures 21-22]. We are reduced to a local question: removing one isolated singular point of the projection $pr_2 : M \times \mathbb{R} \to \mathbb{R}$ by a local isotopy of the smooth structure. Reduce the problem to more and more special cases, apply induction and use the Alexander trick (which as usual highlights the difference between the smooth and PL categories). For [Lur09, Lecture 21, Lemma 3] replace Lurie's proof by the one in [HM74, Theorem 4.3 on p.13]. This talk is a bit technical, but there should be enough time to follow the references closely.

Session 3: Smoothing theory II (November 15, 2012)

Talk 1: Microbundles and their classifying spaces. (Tibor Macko). Develop the theory of microbundles and provide the construction of their classifying space. The speaker may follow [Mil64, §2,3,4,6,7] for the basic definitions and properties. Alternatively start in [Lur09, Lecture 10] just before Definition 6 and continue in [Lur09, Lecture 11]. Give the construction of the classifying space in [Lur09, Lecture 12].

Mention without proof the Kister-Mazur theorem relating microbundles to euclidean bundles [Lur09, Lectures 13,14].

Give one proof of Theorem 0.1 following Theorem 5.12. in [Mil64, §5]

Talk 2: The classification theorem. (Wolfgang Steimle). In this talk first the unstable space version (Theorem 1 in [Lur09, Lecture 16]) of Theorem 0.1 should be proved. A space version means that we will consider the space Smooth(M) of smooth structures on M. The unstable version says that this space is homotopy equivalent to the space $\mathcal{X}_M$ of vector bundle reductions of the unstable PL-tangent microbundle. The speaker should follow the proof in [Lur09, Lectures 15-17]. A corollary is the unstable version of Theorem 0.1 which is then that Smooth(M) is non-empty if and only if $\mathcal{X}_M$ is non-empty.

After this is done, use the PST to obtain Theorem 0.1 from the unstable version [Lur09, Lecture 18]. Another reference for this is [KS77, Essay V].

References for Sessions 2 and 3

[HM74] Morris W. Hirsch and Barry Mazur, *Smoothings of piecewise linear manifolds*, Princeton University Press, Princeton, N. J., 1974. Annals of Mathematics Studies, No. 80. MR0415630 (54 #3711)

[KS77] Robion C. Kirby and Laurence C. Siebenmann, *Foundational essays on topological manifolds, smoothings, and triangulations*, Princeton University Press, Princeton, N.J., 1977. With notes by John Milnor and Michael Atiyah, Annals of Mathematics Studies, No. 88. MR0645390 (58 #31082)

[Lur09] Jacob Lurie, *Topics in geometric topology*, 2009. Lecture notes, available at `http://www.math.harvard.edu/~lurie/937.html` or at `http://www.maths.ed.ac.uk/~aar/surgery/lurie2009.pdf` (as one file).

[Mil64] J. Milnor, *Microbundles. I*, Topology **3** (1964), no. suppl. 1, 53–80. MR0161346 (28 #4553b)

Session 4: Computations of THH (November 29, 2012) organized by Justin Noel

Topological Hochschild homology and cohomology (THH) [Böc86] are the spectrum level analogues of ordinary Hochschild homology and cohomology (HH)[1] Here are some reasons people are interested in $(T)HH$:

(1) It is a homology/cohomology theory on associative rings/ring spectra [Qui67, Sch01].
(2) It is a first approximation to (topological) cyclic homology, which is in turn, an approximation to algebraic K-theory [DGM10].
(3) Its groups appear as the higher homotopy groups of the space of A_∞ structures on a spectrum [Laz03]
(4) It is a source of algebraic invariants that are rich enough to be interesting while simultaneously being computable with existing technology (see the computations of Angeltveit, Ausoni, Gephardt, Hill, and Lawson).

Existing THH calculations inevitably depend on the original computations of $THH^S_*(\mathbb{Z}/p)$ and $THH^S_*(\mathbb{Z})$ due to Bökstedt. Unfortunately these computations have never appeared in print. In this session we will try to present an independent computation of these groups in the former case, which the organizer learned from Vigleik Angeltveit. In the latter case we fall back on the original computation of Bökstedt [Bök85], which is quite similar.

Talk 1: $THH(\mathbb{Z}/p)$ **(Herman Stel).** Below we expand a bit on the arguments presented in [Ang06, pg. 52,59] which can be applied to make some of the computations of [Bök85].

Suppose R is a strictly associative ring spectrum in some symmetric monoidal model category of spectra. For our purposes[2] it suffices to set

$$THH^S(R) := R \wedge_{R^e} R,$$

where $R^e = R \wedge R^{\mathrm{op}}$ [EKMM97, Ch. IX]. Of course, we really want everything in sight to be suitably derived for this to be homotopy invariant. So we should assume that R^e is a cofibrant ring spectrum and R is a cofibrant (or flat) module over R^e.

To compute $THH^S_*(R) = \pi_* R \wedge_{R^e} R$ we use the Tor spectral sequence of [EKMM97, Ch. IV.1]:

$$E^2_{s,t} = \mathrm{Tor}_s^{\pi_* R^e}(\pi_* R, \pi_{*+t} R) \implies THH^S_{s+t}(R).$$

In the case $R = H\mathbb{Z}/p$ we are computing Tor over the dual Steenrod algebra with field coefficients, which is an elementary computation. When $p = 2$ the E_2 term is an exterior algebra on terms in bidegree $(1, 2 \cdot 2^i - 1)$ for $i \geq 0$, and there is no room for differentials, although there is a family of non-trivial multiplicative extensions.

When p is odd, the exterior portion of the dual Steenrod algebra gives rise to a divided power algebra in the E_2 term. Most of the indecomposables from this algebra support a d^{p-1} making the E_p page a truncated polynomial algebra on generators in bidegree $(1, 2 \cdot p^i - 1)$ for $i \geq 0$.

[1] Unfortunately both the homological and cohomological variations admit the same abreviation; the homological variation is distinguished by the use of subscript, as opposed to superscript, indexing.

[2] We do not need a cyclotomic model of THH.

Again every possible non-trivial multiplicative extension is there. In both cases the final answer is

$$THH_*^S(H\mathbb{Z}/p) \cong \mathbb{Z}/p[x],$$

where $|x| = 2$.

Both the differentials in the odd primary case and the multiplicative extensions can be explained by power operations. Our spectral sequence is a bar type spectral sequence and consequently non-trivial Massey product formulae in $\pi_* R^e$ are a source of differentials in spectral sequences of this type (see [McC01, p. 309] for a special (cohomological) case of this). In particular we have the following p-fold Massey product formula

$$\langle \bar{\tau}_i, \cdots, \bar{\tau}_i \rangle = -\beta Q^{p^i} \bar{\tau}_i = -\bar{\xi}_{i+1}$$

[Ang06, pg. 52,59] which is the source of our differentials in the odd primary case.

To show that we obtain a polynomial algebra on one generator, we will show that the generator in total degree 2 is connected to the other classes by iterated pth powers. We give the argument at odd primes, since the even primary case is essentially the same. The power operation formula [BMMS86] tells us $Q^{p^i} Q^{p^{i-1}} \ldots Q^p \bar{\tau}_0 = \bar{\tau}_i$ mod decomposables. The same formula holds after replacing $\bar{\tau}_i$ with $\sigma\bar{\tau}_i$, which are the corresponding elements in our E_2 term, by a naturality argument [Ang06, p. 59]. For degree reasons, the iterated power operation acts by taking iterated pth powers of $\sigma\bar{\tau}_0$ demonstrating the existence of this non-trivial extension.

Talk 2: $THH(\mathbb{Z})$ **(Justin Noel).** In this talk we will follow Bökstedt's proof [Bök85] that

$$THH^S(H\mathbb{Z}) \simeq H\mathbb{Z} \vee \bigvee_{i \geq 1} \Sigma^{2i-1} H\mathbb{Z}/i.$$

First we note that if R is a commutative ring spectrum (E_2 suffices) then $THH^S(R)$ is an R-module. In particular, if R is an Eilenberg-MacLane spectrum associated to a commutative ring then, $THH_S(R)$ is an $H\mathbb{Z}$-module and consequently a wedge of Eilenberg-MacLane spectra.

To compute the rational homotopy groups we smash with $H\mathbb{Q}$, which commutes with the geometeric realization of the standard simplicial spectrum modeling $R \wedge_{R^e} R$. When $R = H\mathbb{Z}$ we obtain a simplicial model for

$$H\mathbb{Q} \wedge_{H\mathbb{Q} \wedge H\mathbb{Q}^{\mathrm{op}}} H\mathbb{Q} \simeq H\mathbb{Q}.$$

As the geometric realization of a simplicial connective spectrum $THH^S(H\mathbb{Z})$ is connective and we see that $THH^S(H\mathbb{Z})$ has one torsion free Eilenberg-Maclane spectrum in degree 0. Now we must determine the torsion summands. We will do this by identifying the mod p homology of this spectrum as comodule over the dual Steenrod algebra and as an algebra.

Recall that

$$\begin{aligned} H\mathbb{Z}/p_* H\mathbb{Z}/p &\cong \mathcal{A}_* \\ H\mathbb{Z}/p_* H\mathbb{Z} &\cong \mathcal{A}_* \square_{\mathcal{A}(0)_*} \mathbb{Z}/p \\ H\mathbb{Z}/p_* H\mathbb{Z}/p^i &\cong \mathcal{A}_* \square_{\mathcal{A}(0)_*} \mathbb{Z}/p \oplus \Sigma \mathcal{A}_* \square_{\mathcal{A}(0)_*} \mathbb{Z}/p \quad (i > 1). \end{aligned}$$

Here $\mathcal{A}(0)_*$ is the exterior algebra on ξ_1. In the last equation the two copies of the homology of $H\mathbb{Z}$ are connected by an ith order Bockstein operation.

The remainder of the talk will require separate arguments for even and odd primes. To prove the result we will identify the homology ring and count summands to determine in which degrees we have a finite Eilenberg-Maclane spectrum. This is not difficult given the results of the previous talk and a clear understanding of the induced map

$$THH^S(H\mathbb{Z}; H\mathbb{Z}/p) \to THH^S(H\mathbb{Z}/p)$$

in homology.

Determining the size of the groups is now equivalent to determining the connecting Bocksteins, which is equivalent to running the Bockstein spectral sequence. By Bökstedt's arguments, using, the filtration of the elements, known relations between power operations and Bocksteins, and the Cartan formula, one can reduce to this to checking a single power operation on a single element is trivial [Bök85, Lemma 1.5]. This vanishes because it is the 'suspension' of a decomposable class.

References for Session 4

[Ang06] Vigleik Angeltveit, *Noncommutative ring spectra*, Ph.D. Thesis, 2006.

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[Böc86] Marcel Böckstedt, *Topological Hochschild homology*, 1986.

[Bök85] Marcel Bökstedt, *The topological Hochschild homology of $\mathbb{Z}$ and $\mathbb{F}_p$*, 1985. Bielefeld.

[DGM10] B. Dundas, T. Goodwillie, and R. McCarthy, *The local structure of algebraic K-theory*, `http://www.uib.no/People/nmabd/b/b.pdf`, 2010.

[EKMM97] A. D. Elmendorf, I. Kriz, M. A. Mandell, and J. P. May, *Rings, modules, and algebras in stable homotopy theory*, Mathematical Surveys and Monographs, vol. 47, American Mathematical Society, Providence, RI, 1997. With an appendix by M. Cole. MR1417719 (97h:55006)

[Laz03] A. Lazarev, *Hoschschild cohomology and moduli spaces of strongly homotopy associative algebras*, Homology Homotopy Appl. **5** (2003), no. 1, 73–100. MR1989615 (2004k:16018)

[McC01] John McCleary, *A user's guide to spectral sequences*, Second, Cambridge Studies in Advanced Mathematics, vol. 58, Cambridge University Press, Cambridge, 2001. MR1793722 (2002c:55027)

[Qui67] Daniel G. Quillen, *Homotopical algebra*, Lecture Notes in Mathematics, No. 43, Springer-Verlag, Berlin, 1967. MR0223432 (36 #6480)

[Sch01] S. Schwede, *Stable homotopy of algebraic theories*, Topology **40** (2001), no. 1, 1–41. MR1791267 (2002d:55026)

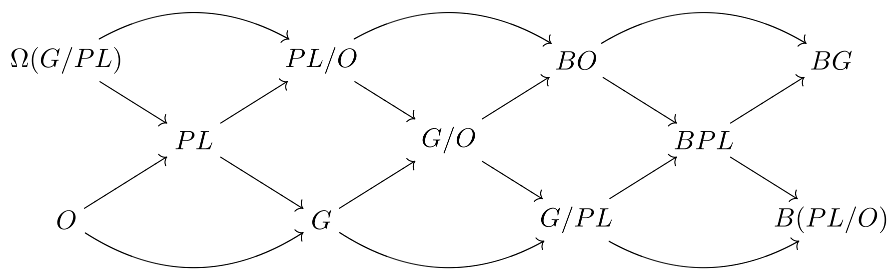
Session 5: The Atiyah-Segal completion theorem (December 13, 2012) organized by Stefan Schwede

Talk 1 by Irakli Patchkoria and Talk 2 by Stefan Schwede.

Session 6: Classifying spaces for surgery (January 10, 2013) organized by Diarmuid Crowley

The aim of these two lectures is to give a survey on what is known and open concerning the spaces and maps appearing in the following braid of infinite loop spaces and infinite loop maps:

(0.1)



Here $O := \lim_{n\to\infty} O(n)$ is the stable orthogonal group, $\mathrm{PL} := \lim_{n\to\infty} \mathrm{PL}(n)$ is the stable group of piecewise linear homeomorphisms of Euclidean space and $G := \lim_{n\to\infty} G(n)$ is the stable monoid of homotopy self equivalences of the sphere.

One motivation to look at this braid is the following: Suppose that M is a PL-manifold with stable tangent micro-bundle classified by a map $M \to BPL$, in the Smoothing Theory lectures of this AG we will see that M admits a smooth structure if and only if the composite $M \to BPL \to B(PL/O)$ is null-homotopic.

The emphasis of these lectures is to review the start of the art, applications to concrete problems and to outline open problems. Hence we plan to give at most sketches of proofs, sometimes just indications of proofs and sometimes no proof. Hence many sources and references will be used. For surgery theory and the identification of the homotopy braid of the above braid with the Kervaire-Milnor braid we will use [Lüc02, Ch. 6.6]. For the homotopy type of G/PL, [Sul05], for the homotopy type of PL/O [May77, Lan00], for the homotopy type of G and G/O [May77], for the map $G \to G/PL$ localised at 2 [BMM73].

Talk 1: The Kervaire-Milnor braid. Let the dimension of manifold n be greater than 4. Begin by using the Generalised Poincaré Conjecture and smoothing theory to identify $\pi_n(\mathrm{PL}/O) \cong \Theta_n$ where Θ_n is the group of oriented diffeomorphism classes of smooth homotopy spheres. Review the Kervaire-Milnor braid from [Lüc02] and sketch the proof that it is isomorphic to the homotopy braid induced by (0.1). During this proof you should consider each of the four homotopy long exact sequences of the braid:

$$\cdots \to \pi_{i+1}(\mathrm{PL}/O) \to \pi_i(O) \to \pi_i(\mathrm{PL}) \to \pi_i(\mathrm{PL}/O) \to \dots \tag{0.2}$$

$$\cdots \to \pi_{i+1}(G/O) \to \pi_i(O) \to \pi_i(G) \to \pi_i(G/O) \to \dots \tag{0.3}$$

$$\cdots \to \pi_{i+1}(\mathrm{PL}/G) \to \pi_i(\mathrm{PL}) \to \pi_i(G) \to \pi_i(G/\mathrm{PL}) \to \dots \tag{0.4}$$

$$\cdots \to \pi_{i+1}(G/\mathrm{PL}) \to \pi_i(\mathrm{PL}/O) \to \pi_i(G/O) \to \pi_i(G/\mathrm{PL}) \to \dots \tag{0.5}$$

For (0.2) state Milnor's result [Mil64] that this sequence splits as a short exact sequences and also Brumfiel's computations of $\pi_i(\mathrm{PL})$ [Bru68, Bru70]. For (0.3) give Adam's results on the J-homomorphism [Ada66] as well as the out-come of the solution of the Adams' Conjecuture. For (0.4) given the computation of $\pi_i(G/\mathrm{PL})$ using the PL surgery exact sequence for the sphere and recall Brumfiel's computation of $\pi_i(\mathrm{PL}/O)$. Identify (0.5) with the surgery long exact sequence of the sphere and review the proof of [KM63] that $\pi_i(\mathrm{PL}/O)$ is finite.

Give the computations of the Kervaire-Milnor braid in dimension $n = 5, \dots, 12$ and interpret this geometrically.

Finally, give the Kirby-Siebenmann result that $TOP/\mathrm{PL} \simeq K(\mathbb{Z}/2, 3)$ [KS77] and hence explain that the above can be repeated with PL replaced by TOP with little change.

Talk 2: $G/\mathrm{PL}_{(2)}$, $G/\mathrm{PL}_{\mathrm{odd}}$, $\mathrm{PL}/O_{\mathrm{odd}}$ and open problems. This talk is a very fast review of a large amount of material.

Begin by stating the equivalences in [May77, Ch V Thm. 4.7 and Thm 4.8]. If possible, define the maps in the equivalences and explain as much of the J-theory diagram in [May77, Ch V] as possible.

Odd-primary results: state the equivalences in [May77, Ch V Theorem 6.8]. Compare with [Lan00, Thm. 7.5]. Give some consequences for computing $[M, \mathrm{PL}/O]_{\mathrm{odd}}$.

2-local results: Give Sullivan's computation of $G/\mathrm{PL}_{(2)}$ as in [MM79, Ch. 4]. As an example compute the structure set of $\mathbb{C}P^n$. Then move onto the map

$$G_{(2)} \to G/\mathrm{PL}_{(2)}$$

and its analysis in [BMM73, Ch. 9]. Give the consequence [BMM73, Thms. 9.6, 9.9 & 9.17] and state some consequences for smoothing theory and surgery theory.

State some open problems regarding the sequence $\mathrm{PL}_{(2)} \to G_{(2)} \to G/\mathrm{PL}_{(2)}$ and the action of π_*^S on $\pi_*(\mathrm{PL}/O)$.

References for Session 6

[Ada66] J. F. Adams, *On the groups $J(X)$. IV*, Topology **5** (1966), 21–71. MR0198470 (33 #6628)

[BMM73] G. Brumfiel, I. Madsen, and R. J. Milgram, PL *characteristic classes and cobordism*, Ann. of Math. (2) **97** (1973), 82–159. MR0310881 (46 #9979)

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[May77] J. Peter May, *E_∞ ring spaces and E_∞ ring spectra*, Lecture Notes in Mathematics, Vol. 577, Springer-Verlag, Berlin, 1977. With contributions by Frank Quinn, Nigel Ray, and Jørgen Tornehave. MR0494077 (58 #13008)

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Session 7: tba (January 24, 2013)
organized by NN