SIMPLY DEFINABLE FAILURES OF WEAK COMPACTNESS

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Abstract. A result of Hung and Negrepontis shows that an uncountable regular cardinal κ is weakly compact if and only if the space ${}^{\kappa}\kappa$, consisting of all functions from κ to κ equipped with the topology whose basic open sets consist of all extensions of partial functions of cardinality less than κ, is not homeomorphic to the subspace ${}^{\kappa}2$ of ${}^{\kappa}\kappa$ consisting of all binary functions. Motivated by recent work of Andretta and Motto Ros, we consider the question whether homeomorphisms of ${}^{\kappa}\kappa$ and ${}^{\kappa}2$ witnessing failures of weak compactness can be simply definable. Our results show that both large cardinals axioms and forcing axioms imply that homomorphisms witnessing that ω_1 is not weakly compact are not simply definable. In contrast, we show that the analogous statements for ω_2 can consistently fail. Finally, we also consider the above questions for successors of singular cardinals and inaccessible cardinals that are not weakly compact.

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