

DESCRIPTIVE SET THEORY AT UNCOUNTABLE CARDINALS: Δ_1^1 -SUBSETS OF ${}^\kappa\kappa$

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ABSTRACT. Let κ be an uncountable regular cardinal with $\kappa = \kappa^{<\kappa}$. A subset of $({}^\kappa\kappa)^n$ is a Σ_1^1 -subset if it is the projection $\rho[T]$ of all cofinal branches through a κ -tree T on κ^{n+1} . We define Σ_k^1 -, Π_k^1 - and Δ_k^1 -subsets of $({}^\kappa\kappa)^n$ as usual.

Given an arbitrary subset A of ${}^\kappa\kappa$, I showed that there is a $< \kappa$ -closed forcing \mathbb{P} that satisfies the κ^+ -chain condition and forces A to be a Δ_1^1 -subset of ${}^\kappa\kappa$ in every \mathbb{P} -generic extension of V . This result allows us to construct a forcing with the above properties that forces the existence of a well-ordering of ${}^\kappa\kappa$ whose graph is a Δ_2^1 -subset of ${}^\kappa\kappa \times {}^\kappa\kappa$. If we also assume $2^\kappa = \kappa^+$, then we can produce a generic well-ordering of ${}^\kappa\kappa$ whose graph is a Δ_1^1 -subset of ${}^\kappa\kappa \times {}^\kappa\kappa$.

In my talk, I want to present the central ideas behind the proofs of these results, focusing on coding subsets of ${}^\kappa\kappa$ by κ -Kurepa trees in forcing extensions and the strong absoluteness properties of this coding.