LOCALLY DEFINABLE WELL-ORDERS

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Abstract. A classical theorem of Mansfield shows that there exists a well-ordering of the set ${}^{\omega}\omega$ of all functions from ω to ω that is definable over the collection $\mathrm{H}(\omega_1)$ of all hereditarily countable sets by a Π_1-formula without parameters if and only if every such function is contained in Gödel's constructible universe L. In particular, the existence of such a well-ordering implies that the continuum hypothesis holds.

We consider the question whether this implication generalizes to higher cardinalities: does the existence of a well-ordering of the set ${}^{\omega_1}\omega_1$ of all functions from ω_1 to ω_1 that is definable over $\mathrm{H}(\omega_2)$ by a Π_1-formula without parameters imply that the GCH holds at ω_1?

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