DEFINABILITY IN MATHEMATICS

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ABSTRACT. The Axiom of Choice implies the existence of a great variety of mathematical objects without providing explicit constructions for these objects and, in many important cases, it can be shown that the existence of these objects is not a consequence of the remaining axioms of set theory. Prominent examples of such objects are non-Lebesgue measurable sets of real numbers, $\mathbb{Q}$-bases of the real numbers and well-orderings of the real numbers.

It is now natural to ask if there is some way to distinguish between such objects and the ones with explicit constructions. Classical results from descriptive set theory provide an answer to this question by showing that the above objects cannot be defined by simple formulas in second-order arithmetic. In my talk, I want to provide an overview of these results and then present newer results that deal with the *set theoretic* definability of the above objects, i.e. their definability in the set theoretic universe $\langle \mathrm{V}, \in \rangle$.

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