THE HEIGHT OF THE AUTOMORPHISM TOWER OF A CENTRELESS GROUP

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Let G be a centreless group. We call a sequence $\langle G_\alpha \mid \alpha \in \mathrm{On}\rangle$ of groups an *automorphism tower of G* if the following statements hold.

(1) $G = G_0$.

(2) For all $\alpha \in \mathrm{On}$, G_α is a normal subgroup of $G_{\alpha+1}$ and the induced map

$$\varphi_\alpha : G_{\alpha+1} \longrightarrow \mathrm{Aut}(G_\alpha);\ h \longmapsto \left[g \mapsto g^h\right]$$

is an isomorphism.

(3) If λ is a limit ordinal, then $G_\lambda = \bigcup_{\alpha<\lambda} G_\alpha$.

It is easy to see that each group G_α is uniquely determined up to isomorphisms that induce the identity map on G.

Simon Thomas showed that for every infinite centreless group G of cardinality κ, there is an $\alpha < (2^\kappa)^+$ with $G_\alpha = G_{\alpha+1}$ and therefore $G_\alpha = G_\beta$ for all $\beta \geq \alpha$. We let $\tau(G)$ denote the minimal ordinal with this property. Given an infinite cardinal κ, we define

$$\tau_\kappa = \mathrm{lub}\{\tau(G) \mid G \textit{ is a centreless group of cardinality } \kappa\}.$$

By Thomas' result, $\tau_\kappa < (2^\kappa)^+$ holds. The following problem is still open.

Problem. *Find a model $\langle M, \in_M\rangle$ of* ZFC *and an infinite cardinal κ in M such that it is possible to* "compute" *the exact value of τ_κ in M.*

Building upon results and methods developed by Itay Kaplan and Saharon Shelah, I want to show how smaller upper bounds for τ_κ can be obtained by combining group-theoretic arguments with *admissible set theory* and *fine structure theory for the inner model* $\mathrm{L}(\mathcal{P}(\kappa))$.

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