THE INFINITE PRODUCTIVITY OF KNASTER PROPERTIES

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ABSTRACT. Given an uncountable regular cardinal κ, we say that a partial order $\mathbb{P}$ is *κ-Knaster* if every set of κ-many conditions in $\mathbb{P}$ contains a subset of cardinality κ consisting of pairwise compatible conditions. This strengthening of the κ-chain condition is typically used because of its nice product behavior: finite support products of κ-Knaster partial orders are κ-Knaster, and the product of a κ-Knaster partial order with a partial order satisfying the κ-chain condition satisfies the κ-chain condition. Moreover, if κ is weakly compact, then the class of κ-Knaster partial orders is closed under ν-support products for every $\nu < \kappa$. This raises the question whether it is possible that the class of κ-Knaster partial orders is closed under countable support products and κ is not weakly compact. I will present results that show that the axioms of ZFC do not answer this question. This is partially joint work with Sean Cox (VCU Richmond).

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