FORCINGS THAT CHARACTERIZE LARGE CARDINALS

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Abstract. The relative consistency of many combinatorial principles in set theory can be established by collapsing a large cardinal to be the successor a smaller cardinal. In many important cases, the resulting principle implies that the relevant successor cardinal has the same large cardinal properties in some canonical inner model and therefore the large cardinal assumption was necessary for the consistency proof. We will consider the question whether certain collapse forcings characterize large cardinal properties through the validity of combinatorial principles in their forcing extensions, in the sense that the collapse forces the principle to hold if and only if the collapsed cardinal possess the corresponding large cardinal property in the ground model.

It is easy to see that the Levy collapse cannot characterize inaccessible cardinals in this way. In contrast, other canonical collapses can characterize many important types of large cardinals. For example, the forcing that first adds a Cohen real and then Levy collapse a cardinal to be ω_2 can characterize inaccessible, Mahlo, weakly compact, indescribable, measurable, supercompact and huge cardinals. In the case of larger large cardinals, these characterizations make use of results of Apter and Hamkins on induced measures in ground models. The characterization of smaller large cardinals relies on classical combinatorial characterizations of these cardinals. Since there are no suitable combinatorial characterizations for indescribable cardinals, the forcing characterizations of these cardinals rely on new results that identify these cardinals as the images of the critical points of certain elementary embeddings.

This is joint work in progress with Peter Holy (Bonn).

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