SMALL MODELS, LARGE CARDINALS, AND INDUCED IDEALS

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Abstract. Despite their central role in modern set theory, large cardinals are still surrounded by many open conceptual questions. In particular, there is no widely accepted formal definition of the intuitive concept of a large cardinal. Instead there are several common ways to formulate large cardinal properties, such as through the existence of elementary embeddings or the validity of partition properties, and for many large cardinal notions equivalent formulations of different types can be found. Yet, there is no commonly accepted framework for their formulation. Moreover, although the linearity of the ordering of large cardinal properties by their consistency strength seems to be deeply woven into their nature, it has not been possible to prove the general validity of this principle and, without a formal definition for the concept of large cardinals, it is not even clear how such an argument should look like.

In my talk, I want to present recent work that addresses the above problems by showing that many large cardinal properties up to measurability can be uniformly characterized through the existence of certain filters for small models of set theory. This correspondence then induces a canonical way to assign ideals to large cardinal notions that coincide with classical large cardinal ideals whenever such ideals had been defined before. It then turns out that, in many important cases, the relations between these ideals reflect the ordering of the corresponding large cardinal properties both under direct implication and consistency strength.

This is joint work with Peter Holy (Udine).

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