

PARTITION PROPERTIES FOR SIMPLY DEFINABLE COLOURINGS

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ABSTRACT. We say that an uncountable regular cardinal κ is Σ_n -weakly compact if every colouring $c : [\kappa]^2 \rightarrow 2$ that is definable by a Σ_n -formula with parameters in $H(\kappa) \cup \{\kappa\}$ has a homogeneous set of cardinality κ . We will discuss results that show that many canonical extensions of ZFC, like large cardinal axioms or forcing axioms, imply that ω_1 is Σ_1 -weakly compact. In contrast, the Σ_1 -weak compactness of ω_2 is not decided by large cardinal axioms and forcing axioms imply that ω_2 is not Σ_1 -weakly compact. We will also discuss inaccessible Σ_1 -weakly compact cardinals, their position in the large cardinal hierarchy and some interesting non-weakly compact examples of such cardinals, e.g. regular limits of measurable cardinals.

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