# SIMPLE FORMULAS DEFINING COMPLICATED SETS

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Abstract. We consider the question whether large cardinal axioms imply that certain complicated sets cannot be defined by simple formulas. More precisely, we ask whether the existence of larger large cardinals is compatible with the existence of a well-ordering of the real numbers that is definable by a $\Sigma_1$-formula that uses a single ordinal as a parameter.

A classical result of Mansfield shows that the existence of a measurable cardinal implies that no well-ordering of the reals can be defined by a $\Sigma_1$-formula that uses a countable ordinal as a parameter. I want to present results showing that the existence of a well-ordering of the reals that is definable by a $\Sigma_1$-formula with parameter $\omega_1$ is compatible with the existence of a Woodin cardinal and incompatible with the existence of a Woodin cardinal with a measurable cardinal above it. Moreover, a similar result holds for $\Sigma_1$-formulas using a Ramsey cardinal as a parameter. Finally, there are analog results for $\Sigma_1$-definitions of the collection of stationary subsets of $\omega_1$ and so-called Bernstein subsets of $\mathcal{P}(\omega_1)$.

This is joint work in progress with Ralf Schindler (Münster) and Philipp Schlicht (Bonn/Münster).

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