ADVANCED MATHEMATICAL LOGIC I (V4A7) SOSE 2024

Tame geometry

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Time and Place. Mondays 2pm ct, Wednesdays 12pm ct We10/Kleiner Hörsaal

E-Campus. https://ecampus.uni-bonn.de/goto_ecampus_crs_3268025.html

Abstract. This course is a first-course in tame geometry, focusing on o-minimality. O-minimality was isolated by van den Dries [vdD84] in order to prove important results from semi-algebraic geometry in this generality, and developed by Pillay and Steinhorn [PS86] as a tameness notion in the setting of dense linear orders. Among the many results that transfer from semi-algebraic geometry to the setting of o-minimal structures, are the monotonicity theorem for definable functions and the cell decomposition theorem.

O-minimality has seen tremendous growth in applications in number theory and algebraic geometry since Pila's proof of the unconditional proof of the André-Oort conjecture in case of products of modular curves [Pil11]. There is now also substantial use of o-minimality in Hogdge theory (see for example Bakker, Klinger and Tsimerman [BKT20] and Klinger's 2022 ICM talk [Kli21]). While we don't have time to cover them here, you will learn all the basics from o-minimality used in these application. Thus this course should also be very useful to students in algebra and geometry who are interested in understanding the application and the role o-minimality plays there.

In this course we will follow the excellent book [vdD98] by van den Dries, covering most of chapters 1-7, and, we will also talk about the Pila-Wilkie counting theorem [PW06], a recent theorem about o-minimal structure that drove the explosion in applications outside of logic.

Prerequisites. Only basic knowledge of first-order logic is assumed. Participation in the course *V3A5/F4A1 - Mathematical Logic* during the Wintersemester 2023/24 is surely helpful, but not required. The script for that course is available on the E-Campus website for this seminar. The course is complemented by the seminar *S4A4 - Graduate Seminar on Logic - O-minimal structures.*

Literature. Lecture notes will be provided. Excellent references are [vdD98] and [Bv22].

References

[BKT20] B. Bakker, B. Klingler, and J. Tsimerman, *Tame topology of arithmetic quotients and algebraicity of Hodge loci*, J. Amer. Math. Soc. **33** (2020), no. 4, 917–939. MR 4155216

[Bv22] Neer Bhardwaj and Lou van den Dries, *On the pila–wilkie theorem*, Expositiones Mathematicae (2022).

[Kli21] Bruno Klingler, *Hodge theory, between algebraicity and transcendence*, 2021.

[Pil11] Jonathan Pila, *O-minimality and the André-Oort conjecture for* $\mathbb{C}^n$, Ann. of Math. (2) **173** (2011), no. 3, 1779–1840. MR 2800724

[PS86] Anand Pillay and Charles Steinhorn, *Definable sets in ordered structures. I*, Trans. Amer. Math. Soc. **295** (1986), no. 2, 565–592. MR 833697

[PW06] J. Pila and A. J. Wilkie, *The rational points of a definable set*, Duke Math. J. **133** (2006), no. 3, 591–616. MR 2228464

[vdD84] Lou van den Dries, *Remarks on Tarski's problem concerning* $(\mathbf{R}, +, \cdot, \exp)$, Logic colloquium '82 (Florence, 1982), Stud. Logic Found. Math., vol. 112, North-Holland, Amsterdam, 1984, pp. 97–121. MR 762106

[vdD98] ———, *Tame topology and o-minimal structures*, London Mathematical Society Lecture Note Series, vol. 248, Cambridge University Press, Cambridge, 1998. MR 1633348