

## GRADUATE SEMINAR ON LOGIC (S4A4) WS 2021/22

### O-minimality

**Instructor.** Philipp Hieronymi ([hieronymi@math.uni-bonn.de](mailto:hieronymi@math.uni-bonn.de))

**Time and Place.** Tuesdays 16.15 -18, Endenicher Allee 60, N 0.007, Hybrid, starts Tuesday October 12th

**Abstract.** Originated in logic, o-minimality has recently seen many applications in number theory and geometry. This seminar serves as an introduction to the construction of o-minimal structures on the real field. The main results discussed in this seminar will be the o-minimality of the following structures:

- $\mathbb{R}_{an}$ , the real field with all restricted analytic function ([1]),
- $\mathbb{R}_{exp}$ , the real field with the exponential function ([7]), and
- $\mathbb{R}_{an,exp}$ , the real field with the exponential function and all restricted analytic functions ([6, 5]).

While the consequences of o-minimality are outlined in [4], the book doesn't contain much about how to prove o-minimality of a given structure and postpones this to a further volume (which never appeared). This seminar covers some of the material of this second volume. Instead of following the original [1], we will read notes by Lou van Dries [3] (which you will receive by email) containing the proof of the o-minimality of  $\mathbb{R}_{an}$ . We will then go through [5] to prove o-minimality of  $\mathbb{R}_{an,exp}$ .

**Prerequisites.** Basic knowledge of first-order logic is definitely needed, but can be picked up easily. While prior knowledge of o-minimality might clarify the motivation why we want to construct o-minimal structures, it is not necessary in order to understand the content of this seminar. A solid background in algebra (in particular, basic valuation theory) is desirable.

#### Talks.

- (1) O-minimality, [4, Chapter 1] (Philipp Hieronymi)
- (2) Semi-algebraic sets, [4, Chapter 2] (Ariadni Ioanna Karanikola, Álvaro Gutiérrez Cáceres), Oct 19 & Oct 26
- (3) Formal power series, [3, p. 5-12] (Varvara Arkhipova), Nov 2
- (4) Convergent power series and Weierstrass preparation, Flatness, [3, p. 13-22] (Juan Sebastian Diaz, Baran Zadeoglu), Nov 9 & Nov 16
- (5) Analytic functions and Weierstrass preparation, [3, p. 23-30] (Sebastian Meyer, Jan Henrik Thomm), Nov 23 & Nov 30
- (6) Quantifier elimination for  $\mathbb{R}_{an}$ , [3, p.30-36] (Lucas Valle Thiele, Alexander Holz), Dec 7 & Dec 14
- (7) An axiomatization for  $T_{an}$ , the theory of  $\mathbb{R}_{an}$ , [5, Section 2] (Felix Jäger), Dec 21
- (8) Valuation theoretic properties of models of  $T_{an}$ , [5, Section 3] (Leo Gitin), Jan 11
- (9) The theory of  $(\mathbb{R}_{an}, \exp)$ , [5, Section 4] (Moritz Hartlieb), Jan 18
- (10) O-minimality and Hardy fields, [5, Section 5] (Gabin Kolly), Jan 25
- (11) Exponentiation is hard to avoid, [2] (Shimal Harichurn), Feb 1

## REFERENCES

1. J. Denef and L. van den Dries, *p-adic and real subanalytic sets*, Ann. of Math. (2) **128** (1988), no. 1, 79–138. MR 951508
2. Chris Miller, *Exponentiation is hard to avoid*, Proc. Amer. Math. Soc. **122** (1994), no. 1, 257–259. MR 1195484
3. Lou van den Dries, *Notes for the next volume of tame topology and o-minimal structures*.
4. ———, *Tame topology and o-minimal structures*, London Mathematical Society Lecture Note Series, vol. 248, Cambridge University Press, Cambridge, 1998. MR 1633348
5. Lou van den Dries, Angus Macintyre, and David Marker, *The elementary theory of restricted analytic fields with exponentiation*, Ann. of Math. (2) **140** (1994), no. 1, 183–205. MR 1289495
6. Lou van den Dries and Chris Miller, *On the real exponential field with restricted analytic functions*, Israel J. Math. **85** (1994), no. 1-3, 19–56. MR 1264338
7. A. J. Wilkie, *Model completeness results for expansions of the ordered field of real numbers by restricted Pfaffian functions and the exponential function*, J. Amer. Math. Soc. **9** (1996), no. 4, 1051–1094. MR 1398816