Catharina Stroppel, Olaf Schnürer Bonn, Sommersemester 2017

SEMINAR ON TENSOR CATEGORIES

Our main reference is [EGNO15].

(1) May 2, Heike Herr: **Coalgebras**
[Swe69, Chapter I and VI]
- category of coalgebras (over a field k), σweedler notation
- examples of coalgebras ([BW03]):
  - kG, $\mathcal{O}(G)$, $U(\mathfrak{g})$, SV, free k-Modul, $k[X]$, $k[X_1, \dots, X_n]$ (two coalgebra structures), matrix coalgebra, divided power coalgebra
  - cofree coalgebra
- $C \mapsto C^*$ and $A \mapsto A^o$ (finite dual coalgebra of an algebra) are adjoint functors
- example of A^o? for $A = k[X]$? linearly recursive sequences
- category of comodules, subcomodules
- examples

(2) May 8, Tim Seynnaeve: **Structure theory for comodules**
[Swe69, Chapter II]
- C-comodules = rational C^*-modules
- compare with G-representations (or rational G-modules) for a linear algebraic group G
- **fundamental theorems** for comodules and coalgebras [Swe69, Thm. 2.1.3]:
  Any finitely generated comodule is finite dimensional.
  Consequence: Any comodule is a filtered union of finite dimensional subcomodules.
  Any finitely generated coalgebra is finite-dimensional.
- $\mathrm{Hom}_{C-\mathrm{CoMod}}(M, C) = \mathrm{Hom}_k(M, k)$ and $\mathrm{Hom}_{C-\mathrm{CoMod}}(M, W \otimes C) = \mathrm{Hom}_k(M, W)$
- $C - \mathrm{CoMod}$ is an abelian category with enough injectives, C is injective cogenerator

(3) May 15, Felix Küng: **Finite type abelian categories and Morita theorems**
[Tak77, Gab62]
- [Tak77, §4]: introduce notion of finite type Grothendieck k-linear abelian category and discuss properties. A k-linear abelian category $\mathcal{A}$ is a finite type Grothendieck k-linear abelian category if:
  - it has has coproducts and filtered colimits are exact (AB5) (if in addition it has a generator then $\mathcal{A}$ is Grothendieck abelian)
  - it has a set of generators of finite length (then $\mathcal{A}$ is locally finite)
  - $\mathcal{A}(M, N)$ is finite dimensional for all objects M, N of finite length
- [Tak77, Theorem 5.1]: A k-linear abelian category $\mathcal{A}$ is of finite type if and only if it is equivalent to the category of comodules over some coalgebra. Moreover, there is a canonical choice for such a coalgebra.
- Given a bicomodule ${}_C M_D$, introduce "cotensor" or "box product" functor $?\Box_C M \colon \mathrm{CoMod}(C) \to \mathrm{CoMod}(D)$ and its left adjoint (taking cohomomorphisms). Give an idea of the Morita theorem [Tak77, Theorem 3.5].

(4) May 22, NN: **The Version of Takeuchi's results in** [EGNO15]
- Deduce the results [EGNO15, Thm. 1.9.15, Thm. 1.10.1] from the results of the previous talk. Note that the notion of a locally finite category is different in [EGNO15]. Discuss pointed coalgebras.
  Maybe helpful: Chin: A brief introduction to coalgebra representation theory; Montgomery.
- [EGNO15, §1.11] Deligne's tensor product of locally finite abelian categories (in the sense of [EGNO15]).

Literatur

[BW03] Tomasz Brzezinski and Robert Wisbauer. *Corings and comodules*, volume 309 of *London Mathematical Society Lecture Note Series*. Cambridge University Press, Cambridge, 2003.

[EGNO15] Pavel Etingof, Shlomo Gelaki, Dmitri Nikshych, and Victor Ostrik. *Tensor categories*, volume 205 of *Mathematical Surveys and Monographs*. American Mathematical Society, Providence, RI, 2015.

[Gab62] Pierre Gabriel. Des catégories abéliennes. *Bull. Soc. Math. France*, 90:323–448, 1962.

[Swe69] Moss E. Sweedler. *Hopf algebras*. Mathematics Lecture Note Series. W. A. Benjamin, Inc., New York, 1969.

[Tak77] Mitsuhiro Takeuchi. Morita theorems for categories of comodules. *J. Fac. Sci. Univ. Tokyo Sect. IA Math.*, 24(3):629–644, 1977.