APPLICATIONS OF STABILITY CONDITIONS TO EXERCISES

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1. T-STRUCTURES AND TORSION PAIRS

Exercise 1. Translate the lemma of 9 to a triangulated category, and prove it!

Exercise 2. Pick a spectral sequence proof in a derived categories textbook and replace it by an argument using the filtration by cohomology.

Exercise 3. Let $\mathcal{T}$ be either the bounded derived cateogory of a smooth projective curve X, or of a quiver without relations. every object in $\mathcal{T}$ is direct sum of its cohomology objects (coherent sheaves, or quiver representations, respectively).

Exercise 4. Consider one of the examples torsion pairs appearing in the lectures. Prove that they are indeed a torsion pair.

Exercise 5.
(1) Prove that if $\mathcal{A}^\sharp \subset \mathcal{D}$ the heart of a bounded t-structure, $A \to B \to C$ is an exact triangle in $\mathcal{D}$ with $A, B \in \mathcal{A}^\sharp$, then the cohomology objects $H^i_\sharp(C)$ C with respect to $\mathcal{A}^\sharp$ can be non-zero only for $i = -1, 0$.
(2) Show that $\mathcal{A}^\sharp$ is an abelian category if we define the kernel of $f\colon A \to B$ to be $H^{-1}_\sharp(\mathrm{cone}\, f)$ and the cokernel to be $H^0_\sharp(\mathrm{cone}\, f)$.
(3) Show that with this definition, any exact triangle $A \to B \to C$ induces a long exact cohomology sequence among the cohomology objects $H^i_\sharp(\underline{\quad})$.

Exercise 6. (*) Consider an elliptic curve E, and its auto-equivalence $\Phi\colon \mathrm{D}^{\mathrm{b}}(E) \to \mathrm{D}^{\mathrm{b}}(E)$ given by the Fourier-Mukai transform of the Poincaré line bundle. Determine the image $\Phi(\mathrm{Coh}\, E)$ of the heart of the standard t-structure.

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